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A contribution to the theory of R&D investments

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Domenico Buccella • Luciano Fanti • Luca Gori

Abstract This research contributes to the theory of cost-reducing R&D investments by offering a tractable three-stage non-cooperative Cournot duopoly game in which R&D-investing firms choose whether to disclose R&D-related information to the rival. Though in a non-cooperative context firms have no incentive to unilaterally disclose information on their cost-reducing R&D activity to prevent the rival from freely appropriate it, this work shows that there is room for the government to design an optimal policy aimed at incentivising unilaterally each owner towards R&D disclosure. Under this welfare improving policy, sharing R&D-related information becomes a Pareto efficient Nash equilibrium strategy of selfish firms. These findings suggest that introducing public subsidies aimed at favouring R&D disclosure represents a win-win result, eliminating the so far established – and unpleasant for both firms and society – non-disclosing outcome.

Keywords Cost-reducing innovation, Nash equilibrium; Government; Social welfare

JEL Classification D43, L13, O31

Declarations of interest None

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1. Introduction

Through the Lisbon Strategy launched in 2000, the European Council defined a crucial target for the EU: «to become the most dynamic and competitive knowledge-based economy in the world by 2010 capable of sustainable economic growth with more and better jobs and greater social cohesion and respect for the environment» (European Commission 2010, p. 2), in which the innovation (and thus the R&D research) is the key element. However, this aim seems to be disregarded as «R&D spending in Europe is below 2%, compared to 2.6% in the US and 3.4% in Japan, mainly as a result of lower levels of private investment» (European Commission 2020, p. 12). Then, the current objective is that of «strengthening knowledge and innovation as drivers of our future growth. This requires improving the quality of our education, strengthening our research performance, promoting innovation and knowledge transfer throughout the Union, making full use of information and communication technologies and ensuring that innovative ideas can be turned into new products and services» (European Commission 2020, pp. 11-12).

Through the Initiative denoted "Innovation Union" aimed «to re-focus R&D and innovation policy on the challenges facing our society», it is claimed the objective «to prioritise knowledge expenditure, including by using tax incentives and other financial instruments to promote greater private R&D investments» (European Commission 2020, pp. 12-13).

Then, it seems to be crucial for Europe to promote private R&D investments and corresponding knowledge spill-overs through the spreading of R&D information by announcing the use of ad hoc public financed incentives.

This article represents a theoretical attempt to investigate the feasibility of a tax policy aimed at reaching the objective of R&D knowledge disclosure by selfish, rational non-cooperative firms belonging to strategic competitive markets. The work contributes to the literature developed by pioneering d'Aspremont and Jacquemin (1988, 1990)'s article, AJ henceforth, building on a model of cost-reducing R&D with (exogenous) spill-overs. It follows Hinloopen (1997, 2000), Milliou (2009) and a recent contribution by Amir et al. (2019), aiming also to be part of the debate on public subsidies to innovation in the pharmaceutical industry and the development of vaccines against COVID-19 following the recent pandemic of the SARS-CoV-2 virus (see, e.g., Xue and Ouellette, 2020; European Commission, 2021; Xie et al., 2021).

Inspired by AJ, this article adds a third disclosure decision stage to a standard two-stage Cournot game in which non-cooperative firms competitively choose both the amount of cost-reducing innovation and output production that should be sold in the market for homogeneous goods. Unlike the absence of public subsidies towards R&D disclosure, in which only the Pareto inefficient Nash equilibrium where firms do not disclose, the present work shows that the public provision of R&D subsidies can dramatically change the perspective of non-cooperative firms letting them be incentivised to spread R&D knowledge as a Pareto efficient outcome of the game, in turn, achieving both the maximisation of social welfare and the main Lisbon Strategy's goal.

The literature following the seminal AJ and related extensions by Henriques (1990) and Suzumura (1992) has developed several interesting trajectories, some of them closely related to our contribution. The first route in this direction is given by the works of Kamien et al. (1992), Ziss (1994), De Bondt (1996), Amir (2000), Amir et al. (2003) and Lambertini and Rossini (2009) that concentrate on cooperative R&D decisions amongst firms competing in the product market, showing that this behaviour is socially beneficial. In this literature spill-overs are exogenous, i.e., a fixed fraction of a firm's R&D investment exogenously flows to competitors, so that each firm has no direct control over the extent of disclosure.

In a second route, the literature assumes that firms endogenously control spill-overs aiming at investigating the owners' decisions about information sharing. These works can be divided into two groups. One group (Poyago-Theotoky, 1999; Atallah, 2004; Lambertini et al., 2004) studies the case in which firms decide on information sharing after they invested in R&D (i.e., spill-overs do not affect the extent of R&D investments). The main result of this branch of literature is to have firms choosing to keep their R&D knowledge secret and thus R&D spill-overs are absent (non-disclosure). The second group (Gersbach and Schmutzler, 2003; Gil-Moltó et al., 2005; Piga and Poyago-Theotoky, 2005; Milliou, 2009) considers the possibility of firms choosing whether sharing R&D outcomes before they invest in R&D (i.e., spill-overs do affect the extent of R&D investments). Protecting or sharing knowledge depends on price or quantity competition (the first paper) on location (second and third papers), and if the extent of R&D spill-overs is not too strong, firms may let R&D knowledge flow to competitors (the last paper). The main result of this branch of literature is to have firms choosing to disclose R&D knowledge with their rivals.

The present article belongs to the literature on exogenous spill-overs, where non-cooperative competitive firms must choose whether to disclose R&D outcomes after they decided on cost-reducing investments. The work explicitly considers the cases of exogenous and optimal policies and offer policy implications that can be applied – amongst others – to the pharmaceutical industry for the development of vaccines against the COVID-19 outbreak.

The remainder of the article is organised as follows. Section 2 outlines the model and discusses the main ingredients of the disclosure decision game. Section 3 studies and discusses the Nash equilibrium outcomes, whereas Section 4 concentrates on optimal R&D subsidisation. Section 5 concludes the article.

2. The model

The economy comprises three types of agents: firms, consumers and the government. The production side is bi-sectorial, i.e., there exist a competitive industry that produces the numeraire good y, and a duopolistic quantity-setting industry where firm i and firm j ($i = \{1,2\}$, $i \neq j$) produce homogeneous goods, q_i and q_j, respectively. Both firms face the perspective of non-cooperatively investing in cost-reducing innovation along the line of the model developed by AJ, augmented later by Bacchiega et al. (2010) and Buccella et al. (2021). Unlike the existing literature, the present work concentrates on the possibility to add another stage to the AJ's two-stage game in which firms non-cooperatively compete in a duopoly by choosing to invest in R&D at the investment stage turning to be Cournot competitors for homogeneous products at the market stage. In the additional stage (the first one), each firm – *after* the decision regarding the extent of cost-reducing R&D investment – chooses whether to disclose R&D-related information (the disclosure decision stage) allowing R&D dissemination at an *exogenous* level of spill-overs (i.e., each firm has no direct control over the extent of the disclosure, e.g., for technological reasons) or keeping R&D knowledge secret.

On the one hand, playing this non-cooperative game in the absence of a public intervention aimed at subsidising external R&D flow leads to the unpleasant Pareto inefficient Nash equilibrium where self-interest and mutual benefit of sharing knowledge do conflict. This is because non-disclosing (i.e., keeping secret) R&D-related results is in the unilateral interest of each non-cooperative firm in the AJ setting. On the other hand, spreading R&D-related information is one of the main aims of the EU's Lisbon Strategy 2020 leading to an increase in consumers' surplus and social welfare.

We now briefly sketch a normalised version of the AJ Cournot duopoly (Buccella et al., 2021) and then summarise the main equilibrium outcomes of the non-cooperative game where firms compete both at the R&D and market stages considering the cases with and without knowledge spill-overs (Table 1 and Table 2, respectively).

Firms i and j choose to invest non-cooperatively in process innovation along the line of d'Aspremont and Jacquemin (1988, 1990) at the first stage of the game by also competing for quantities of homogenous goods in the product market (market stage). There exists a continuum of identical consumers with preferences towards q_i, q_j and y described by a separable utility function $V(q_i, q_j, y): \Re_+^3 \rightarrow \Re_+$, which is linear in the numeraire good y. The representative consumer maximises $V(q_i, q_j, y) = U(q_i, q_j) + y$ with respect to quantities subject to the budget constraint

$pQ + y = M$, where $U(q_i, q_j): \Re_+^2 \rightarrow \Re_+$ is a twice continuously differentiable function, $p > 0$ denotes the market price (representing the marginal willingness to pay of consumers), $Q = q_i + q_j$ is total supply, with q_i and q_j being the quantities produced by firm i and firm j, respectively, and $M > 0$ denotes the representative consumer's exogenous nominal income. This income is high enough to avoid the existence of corner solutions. As $V(q_i, q_j, y)$ is a separable function and it is linear in y there are no income effects on the duopolistic sector. This implies that, for a large enough level of income, the representative consumer's optimisation problem can be reduced to choose the quantities produced by the duopolistic sector to maximise $U(Q) - pQ + M$. Utility maximisation, therefore, yields the inverse demand functions (i.e., the price as a function of quantities): $p = \frac{\partial U}{\partial q_i} = P_i(q_i, q_j)$ for $q_i > 0$, $i = \{1,2\}$, $i \neq j$.

To proceed further with the analysis, we assume explicit demand functions. The linear (inverse) demand $p = a - bQ$, where, respectively, $a > 0$ is a positive parameter representing the market size, and $b > 0$ measures the slope of the market demand being part of its elasticity. This demand structure comes from the usual specification of quadratic utility for consumers' preferences, that is $U(q_i, q_j) = aQ - \frac{b}{2}Q^2$, as proposed by Dixit (1979) and subsequently used, amongst others, by Singh and Vives (1984), Häckner (2000), and Correa-López and Naylor (2004). For reasons of analytical tractability (and without loss of generality), we normalise $a = b = 1$ henceforth. Therefore, the indirect demand for product of firm i is:

$$p = 1 - Q, \, i, j = \{1,2\}, \, i \neq j. \tag{1}$$

The total cost of production and the cost of R&D effort of firm i are respectively given by the functions $C_i(q_i, x_i, x_j)$ and $X_i(x_i)$, where x_i and x_j represent the R&D effort (investment) that firm i and firm j exert, respectively. Following AJ, these functions can be specified through the expressions:

$$C_i(q_i, x_i, x_j) = (w - x_i - \beta_j x_j)q_i, \;\; i, j = \{1,2\}, i \neq j, \tag{2}$$

and

$$X_i(x_i) = \frac{g}{2}x_i^2, \;\; i, j = \{1,2\}, i \neq j, \tag{3}$$

where $g > 0$ is a parameter measuring R&D efficiency. It scales up/down R&D investment total costs and represents an exogenous index of technological progress measuring, for example, the appearance of a new, cost-effective technology, weighting the degree at which the available technology for process innovation affects investment decisions and firm's profits. A reduction in g can be interpreted as a technological advance so that investing in R&D becomes cheaper (i.e., the efficiency of R&D investment increases). In addition, $\beta_j \in [0,1]$ captures the extent of spill-overs (externality) of the R&D investment activity of firm j exogenously flowing as a cost-reducing device towards firm i (i.e., the amount of information that firm j exogenously discloses). We assume that both firms share this characteristic of the extent of technological spill-overs: firm i discloses at the rate β_i towards firm j, and firm j discloses at the rate β_j towards firm i.

This scenario represents the standard case of exogenous spill-overs – with respect to which a fixed fraction of a firm's R&D process innovation exogenously flows to competitors, so that each firm has no direct control over the extent of disclosure for, e.g., technological reasons – and directly follows AJ and the subsequent contributions by Henriques (1990), Suzumura (1992), Kamien et al. (1992), De Bondt (1996) and Bacchiega et al. (2010). When $\beta_j = 0$ there are no R&D externalities, resembling the case of non-disclosure of R&D information from firm j to firm i. When $\beta_j = 1$, R&D information produced by firm j is fully shared with firm i, so that R&D disclosure is at its (exogenous) highest intensity.

The expression representing the firm's technology in Eq. (2) implies that the unitary cost of production should be positive so that $w - x_i - \beta_j x_j > 0$ must hold for any β_j, where $0 < w < 1$ measures the unitary technology of production cost irrespective of R&D investments. Moreover, the expression representing the cost of R&D effort in Eq. (3) reveals diminishing returns in the R&D

technology exerted by firm i. Therefore, each firm sustains the cost of R&D effort with a technology displaying decreasing returns to scale to achieve the benefit of reducing the total unit costs of production with constant returns to scale.

By using Eqs. (1), (2) and (3) the profit function of firm i can be written as follows:

$$\Pi_i = (1 - Q)q_i - \left(w - x_i - \beta_j x_j\right)q_i - \frac{g}{2}x_i^2. \quad (4)$$

Table 1 and Table 2 respectively summarise the equilibrium outcomes obtained by the investing firms in both cases they are disclosing at the exogenous rate $\beta_i = \beta_j = \beta > 0$ and they are not disclosing, i.e., $\beta_i = \beta_j = \beta = 0$. The feasibility conditions following Table 1 (see Buccella et al., 2021 for a thoughtful analysis on this issue) are $g > \frac{2(1+\beta)(2-\beta)}{9}$ (the stability condition that holds when x_i and x_j are strategic complements) and $g > \frac{2(2-\beta)^2}{9}$ (the second-order condition). The feasibility conditions following Table 2 hold accordingly by setting $\beta = 0$.

Table 1. Equilibrium outcomes in the AJ model when both firms disclose ($\beta_i = \beta_j = \beta > 0$).

$x^*(\beta)$	$\frac{2(1-w)(2-\beta)}{9g-2(1+\beta)(2-\beta)}$
$q^*(\beta)$	$\frac{3g(1-w)}{9g-2(1+\beta)(2-\beta)}$
$p^*(\beta)$	$\frac{3g(1+2w)-2(1+\beta)(2-\beta)}{9g-2(1+\beta)(2-\beta)}$
$\Pi^*(\beta)$	$\frac{g(1-w)^2[9g-2(2-\beta)^2]}{[9g-2(1+\beta)(2-\beta)]^2}$
$CS^*(\beta)$	$\frac{18g^2(1-w)^2}{[9g-2(1+\beta)(2-\beta)]^2}$
$W^*(\beta)$	$\frac{4g(1-w)^2[9g-(2-\beta)^2]}{[9g-2(1+\beta)(2-\beta)]^2}$

Table 2. Equilibrium outcomes in the AJ model when both firms do not disclose ($\beta_i = \beta_j = 0$).

$x^*(0)$	$\frac{4(1-w)}{9g-4}$
$q^*(0)$	$\frac{3g(1-w)}{9g-4}$
$p^*(0)$	$\frac{3g(1+2w)-4}{9g-4}$
$\Pi^*(0)$	$\frac{g(1-w)^2(9g-8)}{(9g-4)^2}$
$CS^*(0)$	$\frac{18g^2(1-w)^2}{(9g-4)^2}$
$W^*(0)$	$\frac{4g(1-w)^2}{9g-4}$

Comparison of Table 1 and Table 2 easily reveals that $W^*(\beta) > W^*(0)$. Therefore, in line with the Lisbon's Strategy the circulation of R&D information is beneficial for society as it contributes to increasing social welfare (W^*) by increasing consumers' surplus (CS^*) and firms' profits (Π^*).[1] However, adding the disclosure decision stage to AJ's two-stage game leads to the Pareto inefficient Nash equilibrium where both selfish firms choose to avoid disclosure of R&D outcomes (prisoner's dilemma). Thus, although the circulation of information is beneficial for the whole society, it cannot be achieved through the non-cooperative choices of selfish innovating firms in a strategic context. This is because each firm unilaterally prefers not to disseminate the results of its R&D in order not to favour its rival by reducing its production costs, i.e., each firm does not want to share information on its cost-reducing R&D for preventing the rival from freely appropriate it.

To overcome this lacuna, we follow the literature on R&D subsidies (Hinloopen 1997; Hinloopen 2000; Amir et al., 2019) and take Milliou (2009) seriously by introducing a public policy aimed at incentivising R&D disclosure in a non-cooperative game by reducing the cost of R&D effort of each firm (along the line of Amir et al., 2019). Specifically, the R&D subsidies towards firm i ($\Sigma_i > 0$) and firm j ($\Sigma_j > 0$) are financed at a balanced budget with a uniform non-distorting lump-sum tax ($T > 0$) on the side of consumers (i.e., the tax does not change the consumer's marginal rate of substitution). The available post tax exogenous nominal income of the representative consumer ($M - T > 0$) is high enough to avoid corner solutions. Therefore, the government budget constraint reads as follows:

$$T = \Sigma_i + \Sigma_j, \tag{5}$$

where $\Sigma_i = \sigma\beta_i \frac{g}{2} x_i^2$, $\Sigma_j = \sigma\beta_j \frac{g}{2} x_j^2$ and $\sigma \geq 0$ is the subsidy rate.

Definitively, the government first announces the policy and then firms are engaged in a three-stage non-cooperative *disclosure decision game* with complete information in which each of them must choose whether to disclose R&D-related information (at an exogenous rate) to the rival at stage one (*the disclosure decision stage*). At stage two (*the R&D stage*), firms compete on the extent of process R&D investment in a standard AJ setting. Finally, at stage three (*the market stage*), they compete on quantities in the product market. As usual, the game is solved by adopting the backward induction logic. Therefore, firms decide on information sharing after they invested in R&D, along the line of Poyago-Theotoky (1999), Atallah (2004) and Lambertini et al. (2004). Unlike them, however, we concentrate on the case of exogenous spill-overs and let them play non-cooperatively the disclosure decision game.

3. The disclosure decision game

This section initiates the study of the Cournot rivalry non-cooperative R&D disclosure decision game to analyse the individual incentive to disclose (or not to disclose) by first considering the symmetric subgame in which the government subsidises R&D disclosure at the rate $\sigma \geq 0$ – along the line of the policy discussed so far – and both firms disclose (Section 3.1). Second, it moves to the symmetric subgame in which both firms do not disclose, so that $\beta_i = \beta_j = \beta = 0$ (Section 3.2). Third, it discusses the asymmetric subgame in which only one firm (say, firm i) chooses to disclose (Section 3.3), and finally it adds the disclosure decisions stage by analysing the emergence of a range of Nash equilibria (Section 3.4).

3.1. The symmetric subgame in which firms disclose (D/D)

Consider the possibility that firm i (resp. firm j) chooses to disclose R&D-related information to the rival thus sharing with the competitor its R&D outcome at the rate $\beta_i > 0$ (resp. $\beta_j > 0$), knowing

[1] This can easily be shown by computing $\left.\frac{\partial CS^*(\beta)}{\partial \beta}\right|_{\beta=0} = \frac{72\ (1-w)^2}{(9g-4)^3} > 0$, $\left.\frac{\partial \Pi^*(\beta)}{\partial \beta}\right|_{\beta=0} = \frac{4g(1-w)^2(27g-1\ )}{(9g-4)^3} > 0$ for any g that satisfies the feasibility conditions of the model, and $\left.\frac{\partial W^*(\beta)}{\partial \beta}\right|_{\beta=0} = \frac{32\ (1-w)^2}{(9g-4)^2} > 0$.

also that the government is financing R&D disclosure at the rate $\sigma \geq 0$. Therefore, the profit function of firm i can be written as follows:

$$\Pi_i^{D/D} = (1-Q)q_i - \left(w - x_i - \beta_j x_j\right)q_i - \frac{g}{2}x_i^2(1-\beta_i\sigma), \quad (4)$$

where β_j captures the extent of R&D spillovers exogenously flowing from firm j (as a cost reducing device) to firm i, $\sigma < \frac{1}{\beta_i}$ (to preserve the incentive to invest in R&D) and the upper script D/D stands for R&D disclosure of both firms. As is clear from Eq. (4), sharing R&D outcomes generates (net of the equilibrium feedback effects) benefits and costs to each firm: it contributes to reduce the total cost of production of the rival, thus increasing its incentive to free ride on existing knowledge avoiding, in turn, investing itself in R&D; it also reduces the extent of its R&D effort through the public subsidy.

At the market stage of the game, each firm chooses the amount of output to maximise profits. Maximisation of (4) with respect to q_i leads to the following downward-sloping reaction function of firm i in the (q_i, q_j) space as a function also of R&D efforts x_i and x_j, that is:

$$\frac{\partial \Pi_i^{D/D}}{\partial q_i} = 0 \Leftrightarrow \overline{q}_i^{D/D}\left(q_j, x_i, x_j\right) = \frac{1-w-q_j+x_i+\beta x_j}{2}. \quad (5)$$

Using Eq. (5) together with the counterpart for firm j allows us to obtain the system of output reaction functions that depend on the R&D effort. The solution of the system of output reaction functions $\overline{q}_i^{D/D}\left(q_j, x_i, x_j\right)$ $(i = \{1,2\},\ i \neq j)$ allows us to get the following equilibrium output obtained at the third stage of the symmetric subgame D/D:

$$\overline{\overline{q}}_i^{D/D}\left(x_i, x_j\right) = \frac{1-w+(2-\beta)x_i+(2\beta-1)x_j}{3}. \quad (6)$$

Eq. (13) shows that output production of firm i depends on its R&D investment (due to a twofold reason) as well as on the R&D investment carried out by firm j (due to the R&D externality). On one hand, the R&D investment undertaken by firm i allows for a direct increase in the amount of its output production (whose intensity is weighted by the coefficient 2) due to the strategic interaction with the rival. Therefore, firm i increases production through this channel. On the other hand, there exists a mitigating effect of x_i on q_i because of the amount of R&D-related information externalities flowing from firm i to firm j (whose intensity is weighted by the coefficient β). Therefore, firm i reduces production through this channel. However, the strength of the latter effect can never counterbalance the strength of the former, including under the most extreme conditions, i.e., when disclosure is at its maximum intensity ($\beta = 1$). Definitively, an increase in x_i always causes an increase in q_i. Additionally, the R&D disclosure of both firms allows for a further increase in output production of firm i through the R&D investments of firm j if and only if the extent of technological spill-overs is sufficiently large ($\beta > \frac{1}{2}$, i.e., x_i and x_j are strategic complements), turning otherwise to a smaller extent if the technological spill-overs is sufficiently small ($\beta < \frac{1}{2}$, i.e., x_i and x_j are strategic substitutes). Therefore, increasing market share in the product market needs a high rate of disclosure.

Substituting out Eq. (6) together its counterpart for firm j in Eq. (4) allows to obtain firm i's profits as a function of x_i and x_j, i.e., $\Pi_i^{D/D}(x_i, x_j)$. Therefore, the maximisation of firm i's profits with respect to x_i yields:

$$\frac{\partial \Pi_i^{D/D}(x_i,x_j)}{\partial x_i} = 0 \Leftrightarrow \overline{x}_i^{D/D} = \frac{2(2-\beta)\left[1-w+(2\beta-1)x_j\right]}{9g(1-\beta\sigma)+2\beta(4-\beta)-8}. \quad (7)$$

Using Eq. (7) together with the corresponding counterpart for firm j allows to get the system of R&D reaction functions in the space (x_i, x_j). Solving this system allows us to obtain the amount of equilibrium investment (denoted as usual with an asterisk) following the process innovation effort of firm i at the second stage of the game under D/D (and consequently the symmetrical firm j's response), that is:

$$x_i^{*D/D} = \frac{2(1-w)(2-\beta)}{9g(1-\beta\sigma)-2(1+\beta)(2-\beta)}. \tag{8}$$

From Eq. (8), $x_i^{*D/D} > 0$ if and only if the denominator is positive, that is $g > \frac{2(1+\beta)(2-\beta)}{9(1-\beta\sigma)} := g_{SC}^{\beta_{high}}(\beta,\sigma)$, as will be clear from Eq. (12), where the subscript SC denotes "Stability Condition".

The second-order condition for a maximum (concavity) requires that $\frac{\partial^2 \Pi_i^{D/D}(x_i,x_j)}{\partial x_i^2}\Big|_{x_i=x_i^{*D/D}} < 0$. This implies that the inequality

$$g > \frac{2(2-\beta)^2}{9(1-\beta\sigma)} := g_{SOC}^{D/D}(\beta,\sigma) \text{ (second-order condition)}, \tag{9}$$

must hold to guarantee that the solution to the profit maximisation problem is meaningful, where the subscript SOC denotes "Second Order Condition". The R&D equilibrium characterised by the expression in (8) is stable (in the sense of Seade, 1980) if and only if the reaction functions defined in the R&D space should cross adequately (Henriques, 1990). If the reaction functions are downward-sloping (resp. upward-sloping), x_i and x_j are strategic substitutes (resp. complements). This holds when the R&D externality is small (resp. large). The stability conditions require that $\left|\frac{dx_i}{dx_j}\right| < 1$ in both cases of strategic substitutability and complementarity thus leading to a relationship between g, β, as in Henriques (1990), Bacchiega et al. (2010) and Buccella et al. (2021), and σ. Formally, by computing the derivative:

$$\frac{dx_i}{dx_j} = \frac{2(2\beta-1)(2-\beta)}{9g(1-\beta\sigma)-2(2-\beta)^2}, \tag{10}$$

one can see that the denominator is positive if and only if $g > g_{SOC}(\beta,\sigma)$, which should always be fulfilled for concavity for any $0 < \beta \leq 1$ and $\sigma < \frac{1}{\beta}$. Therefore, $\frac{dx_i}{dx_j} < 0$ if and only if $\beta < \frac{1}{2}$ (x_i and x_j are strategic substitutes) and $\frac{dx_i}{dx_j} > 0$ if and only if $\beta > \frac{1}{2}$ (x_i and x_j are strategic complements). The stability conditions $\left|\frac{dx_i}{dx_j}\right| < 1$ in the R&D model with public subsidy require that one should impose:

$$g > \frac{2(1-\beta)(2-\beta)}{3(1-\beta\sigma)} := g_{SC}^{D/D,\beta_{low}}(\beta,\sigma) \text{ if } 0 < \beta < \tfrac{1}{2}, \text{ (x_i and x_j are strategic substitutes)}, \tag{11}$$

and

$$g > \frac{2(1+\beta)(2-\beta)}{9(1-\beta\sigma)} := g_{SC}^{D/D,\beta_{high}}(\beta,\sigma) \text{ if } \tfrac{1}{2} < \beta \leq 1, \text{ (x_i and x_j are strategic complements)}, \tag{12}$$

where $g_{SC}^{D/D,\beta_{low}}(\beta,\sigma) \geq g_{SC}^{D/D,\beta_{high}}(\beta,\sigma)$ for any $0 < \beta \leq \frac{1}{2}$ and $\sigma < \frac{1}{\beta}$, and $g_{SC}^{D/D,\beta_{low}}(\beta,\sigma) < g_{SC}^{D/D,\beta_{high}}(\beta,\sigma)$ for any $\frac{1}{2} < \beta \leq 1$ and $\sigma < \frac{1}{\beta}$. Therefore, the condition that guarantees positive R&D investments ($g > g_{SC}^{D/D,\beta_{high}}(\beta,\sigma)$) from Eq. (8) and the second-order condition ($g > g_{SOC}^{D/D}(\beta,\sigma)$) are fulfilled for any $0 < \beta \leq 1$ and $\sigma < \frac{1}{\beta}$ if the stability conditions are satisfied. It is also important to pinpoint that an increase in σ increases the incentive of firm i to invest in R&D, that is $\frac{\partial x_i^{*D/D}}{\partial \sigma} = \frac{18g\beta(1-w)(2-\beta)}{[9g(1-\beta\sigma)-2(1+\beta)(2-\beta)]^2} > 0$, whereas a change in β when the government provides subsidies to favour R&D disclosure implies that

$$\frac{\partial x_i^{*D/D}}{\partial \beta} = \frac{-2(1-w)[9g(1-2\sigma)-2(2-\beta)^2]}{[9g(1-\beta\sigma)-2(1+\beta)(2-\beta)]^2}. \tag{13}$$

As the denominator of (13) is always positive, by looking at its numerator one can see that $\frac{\partial x_i^{*D/D}}{\partial \beta} > 0$ for any β and g if $\sigma \geq \frac{1}{2}$. When the subsidy rate is sufficiently large, each firm has an incentive to increase the amount of cost-reducing R&D investment irrespective of the relative size of the externality resulting from the rival's investment. Differently, when the subsidy rate is sufficiently low ($\sigma < \frac{1}{2}$), the effect on $x_i^{*D/D}$ of a change in β is always negative. In fact, if $g > g_{SOC}^{D/D}(\beta,\sigma) > g_{SOC}^{D/D}(\beta,\sigma)\frac{1-\beta\sigma}{1-2\sigma}$, then the term in brackets is positive and thus $\frac{\partial x_i^{*D/D}}{\partial \beta} < 0$, and the case $g < g_{SOC}^{D/D}(\beta,\sigma)\frac{1-\beta\sigma}{1-2\sigma}$ cannot hold as $g > g_{SOC}^{D/D}(\beta,\sigma)$ must be fulfilled for concavity. This implies that providing public subsidies at a relatively low rate is not sufficient to avoid the free riding of each firm on the R&D activity of the rival, so that each firm has an incentive to reduce its R&D effort to avoid the rival from freely appropriate it.

The analysis made so far should be augmented with additional constraints on the side of the costs of production. As known from Eq. (2), the unitary production cost $w - x_i - \beta x_j$ must be positive. Therefore, by using Eq. (8) the inequality $w - x_i - \beta x_j > 0$ is fulfilled if and only if:

$$g > \frac{2(1+\beta)(2-\beta)}{9w(1-\beta\sigma)} \coloneqq g_T^{D/D}(\beta,\sigma,w), \text{ (R\&D cost condition)}, \tag{14}$$

where the subscript T stands for "Threshold".

The inequality in (14) must hold as an additional threshold in determining meaningful Nash equilibrium outcomes of the game, as will be clear from the analysis presented in Section 3.4. The shape of the threshold $g_T^{D/D}(\beta,\sigma,w)$ also depends on w. Therefore, it is important to study the conditions under which the threshold $g_T^{D/D}(\beta,\sigma,w)$ is binding compared to the stability conditions for the subgame D/D.

Comparison of (12) and (14) reveals that $g_T^{D/D}(\beta,\sigma,w) > g_{SC}^{D/D,\beta_{hig}}(\beta,\sigma)$ for any $w < 1$ and $g_T^{D/D}(\beta,\sigma,1) \to g_{SC}^{D/D,\beta_{high}}(\beta,\sigma)$ from above for $w \to 1$. Differently, comparison of (11) and (14) reveals that $g_T^{D/D}(\beta,\sigma,w)$ can be higher or lower than $g_{SC}^{D/D,\beta_{low}}(\beta,\sigma)$ depending on the relative size of β, σ and w. Proposition 1 deepens this result by showing that $g_T^{D/D}(\beta,\sigma,w)$ can be binding in the (β, g) space depending on some conditions on the main parameters of the problem. Let us first define $\beta_T^{D/D} \coloneqq \frac{3w-1}{1+3w}$ as a threshold value of the intensity of the R&D externality such that $g_T^{D/D}(\beta,\sigma,w) = g_{SC}^{\beta_{low}}(\beta,\sigma)$ in the (β, g) space. Then, $\beta_T^{D/D} \to \frac{1}{2}$ if $w \to 1$ and $\beta_T^{D/D} < \frac{1}{2}$ for any $w < 1$. In addition, $\beta_T^{D/D} < 0$ if $w < \frac{1}{3}$ and $\beta_T^{D/D} > 0$ if $w > \frac{1}{3}$. Then, the following proposition holds.

Proposition 1. 1) If $w < \frac{1}{3}$ then $g_T^{D/D}(\beta,\sigma,w)$ is binding in the (β, g) space for any $0 < \beta \leq 1$ and $\sigma < \frac{1}{\beta}$ for the subgame D/D. 2) If $w > \frac{1}{3}$ then 2.1) $g_{SC}^{D/D,\beta_{low}}(\beta,\sigma)$ is binding in the (β, g) space for any $\beta < \beta_T^{D/D}$ for the subgame D/D, and 2.2) $g_T^{D/D}(\beta,\sigma,w)$ is binding in the (β, g) space for any $\beta > \beta_T^{D/D}$ for the subgame D/D. 3) If $w \to 1$ then $g_{SC}^{D/D,\beta_{low}}(\beta,\sigma)$ is binding for any $0 < \beta < \frac{1}{2}$ and $\sigma < \frac{1}{\beta}$, and $g_{SC}^{D/D,\beta_{high}}(\beta,\sigma)$ is binding for any $\frac{1}{2} < \beta \leq 1$ and $\sigma < \frac{1}{\beta}$ for the subgame D/D.

Proof. 1) If $w < \frac{1}{3}$ then $\beta_T^{D/D} < 0$ and $g_T^{D/D}(\beta,\sigma,w) > g_{SC}^{D/D,\beta_{low}}(\beta,\sigma)$ for any $0 < \beta \leq 1$ and $\sigma < \frac{1}{\beta}$. 2) If $w > \frac{1}{3}$ then $\beta_T^{D/D} < 0$ and 2.1) $g_T^{D/D}(\beta,\sigma,w) \leq g_{SC}^{D/D,\beta_{low}}(\beta,\sigma)$ for any $0 < \beta \leq \beta_T^{D/D}$ and $\sigma < \frac{1}{\beta}$, and 2.2) $g_T^{D/D}(\beta,\sigma,w) > g_{SC}^{D/D,\beta_{low}}(\beta,\sigma)$ for any $\beta_T^{D/D} < \beta \leq 1$ and $\sigma < \frac{1}{\beta}$. 3) If $w \to 1$ then $\beta_T^{D/D} \to \frac{1}{2}$ and $g_T^{D/D}(\beta,\sigma,1) \to g_{SC}^{D/D,\beta_{high}}(\beta,\sigma)$ from above for any $0 < \beta \leq 1$ and $\sigma < \frac{1}{\beta}$. Therefore, $g_{SC}^{D/D,\beta_{low}}(\beta,\sigma) \geq g_{SC}^{D/D,\beta_{hig}}(\beta,\sigma)$ for any $0 < \beta \leq \frac{1}{2}$ and $\sigma < \frac{1}{\beta}$, and $g_{SC}^{D/D,\beta_{low}}(\beta,\sigma) < g_{SC}^{D/D,\beta_{high}}(\beta,\sigma)$ for any $\frac{1}{2} < \beta \leq 1$ and $\sigma < \frac{1}{\beta}$. **Q.E.D.**

We now move forward by continuing the equilibrium analysis of the subgame D/D. By using the symmetrical equilibrium R&D expression in (8) and substituting out for $x_i^{*D/D}$ in the equilibrium output obtained at the third stage of the game, one gets the amount of output produced by firm i ($i = \{1,2\}$, $i \neq j$) at equilibrium under D/D, that is:

$$q_i^{*D/D} = \frac{3g(1-w)(1-\beta\sigma)}{9g(1-\beta\sigma)-2(1+\beta)(2-\beta)}. \tag{15}$$

Eq. (15) reveals that $g > g_{SC}^{D/D,\beta_{high}}(\beta,\sigma)$ is sufficient to guarantee a positive output production for both firms. In addition, from the expressions in (8) and (15) one can easily get the equilibrium values of the market price of product of variety i and profits of firm i, which are respectively given by the following equations:

$$p_i^{*D/D} = \frac{3g(1-\beta\sigma)(1+2w)-2(1+\beta)(2-\beta)}{9g(1-\beta\sigma)-2(1+\beta)(2-\beta)}, \tag{16}$$

where the denominator is positive if $g > g_{SC}^{D/D,\beta_{high}}(\beta,\sigma)$, and

$$\Pi_i^{*D/D} = \frac{g(1-w)^2(1-\beta\sigma)\left[9g(1-\beta\sigma)-2(2-\beta)^2\right]}{[9g(1-\beta\sigma)-2(1+\beta)(2-\beta)]^2}. \tag{17}$$

The expressions of the equilibrium price and equilibrium profits in (16) and (17) reveal that $p_i^{*D/D} > 0$ if $g > \frac{2(1+\beta)(2-\beta)}{3g(1-\beta\sigma)(1+2w)} \coloneqq g_p^{D/D}(\beta,\sigma,w)$ and $\Pi_i^{*D/D} > 0$ if $g > g_{SOC}^{D/D}(\beta,\sigma)$. We note that $g_p^{D/D}(\beta,\sigma,w) < g_{SC}^{D/D,\beta_{high}}(\beta,\sigma)$ for any $0 < w < 1$ and $g_p^{D/D}(\beta,\sigma,1) \to g_{SC}^{D/D,\beta_{high}}(\beta,\sigma)$ if $w \to 1$. Therefore, both thresholds are satisfied if either $g > g_T^{D/D}(\beta,\sigma,w)$ or $g > g_{SC}^{D/D,\beta_{low}}(\beta,\sigma)$ holds.

The equilibrium values of consumers' surplus ($CS^{*D/D}$) and producers' surplus ($PS^{*D/D}$) that can be obtained in the D/D subgame are summarised as follows:

$$CS^{*D/D} = \frac{1}{2}\left(q_i^{*D/D} + q_j^{*D/D}\right)^2 = \frac{18g^2(1-w)^2(1-\beta\sigma)^2}{[9g(1-\beta\sigma)-2(1+\beta)(2-\beta)]^2}. \tag{18}$$

and

$$PS^{*D/D} = \Pi_i^{*D/D} + \Pi_j^{*D/D} = \frac{2g(1-w)^2(1-\beta\sigma)\left[9g(1-\beta\sigma)-2(2-\beta)^2\right]}{[9g(1-\beta\sigma)-2(1+\beta)(2-\beta)]^2}. \tag{19}$$

Therefore, equilibrium social welfare under D/D, $W^{*D/D}$, is given by:

$$W^{*D/D} = CS^{*D/D} + PS^{*D/D} - T^{*D/D} = \frac{4g(1-w)^2[9g(1-\beta\sigma)^2-(2-\beta)^2]}{[9g(1-\beta\sigma)-2(1+\beta)(2-\beta)]^2}. \tag{20}$$

Section 2 already showed that social welfare when both firms disclose ($\beta > 0$) is larger than social welfare when both firms do not disclose ($\beta = 0$). However, the non-disclosing outcome is in the unilateral interest of every profit-maximising firm in the absence of R&D subsidies. This section augments the result showing the existence of an optimal policy (Proposition 2). Later (Section 3.4), the article will show that sharing R&D-related information when the government provides subsidies to firms (at a balanced budget) to favour R&D disclosure can become a (Pareto efficient) sub-game perfect Nash equilibrium of the game.

The analysis of the expression in (20) leads to following proposition.

Proposition 2. The introduction of R&D subsidies to incentivise R&D disclosure is welfare improving and there exists a welfare-maximising optimal policy if and only if:

$$\sigma = \sigma^{OPT} := \frac{3}{2(1+\beta)}, \tag{21}$$

where $\sigma^{OPT} < \frac{1}{\beta}$, $\sigma^{OPT} \to \frac{3}{2}$ if $\beta \to 0$ and $\sigma^{OPT} = \frac{3}{4}$ if $\beta = 1$.

Proof. Differentiating the expression in (20) with respect to σ one gets:

$$\left.\frac{\partial W^{*D/D}}{\partial \sigma}\right|_{\sigma=0} = \frac{216g^2\beta^2(2-\beta)(1-w)^2}{[9g-2(1+\beta)(2-\beta)]^3} > 0, \tag{22}$$

and

$$\frac{\partial W^{*D/D}}{\partial \sigma} = \frac{72g^2\beta^2(2-\beta)(1-w)^2[3-2(1+\beta)\sigma]}{[9g(1-\beta\sigma)-2(1+\beta)(2-\beta)]^3} = 0 \Leftrightarrow \sigma = \sigma^{OPT}. \tag{23}$$

Therefore, $\frac{\partial W^{*D/D}}{\partial \sigma} > 0$ if $\sigma < \sigma^{OPT}$ and $\frac{\partial W^{*D/D}}{\partial \sigma} < 0$ if $\sigma > \sigma^{OPT}$. **Q.E.D.**

Proposition 2 resembles Amir et al. (2019) and has a relevant policy application, in line with the EU's Lisbon Strategy 2020. Public policies aiming at subsiding R&D spillovers generate a clear trade-off between consumers' surplus and producers' surplus. On the consumer side, the policy provides benefits by allowing an increase in output production. On the producer side, the effect of the policy is a priori uncertain as it may increase firms' profits until the percentage increase in output production is larger than the subsequent percentage reduction in the market price that consumers are willing to pay (the market demand is downward sloping). However, profits may reduce if the percentage reduction in the market price is large enough to more than compensate for the positive effects on profits due to the increase in output production. If marginal benefits (as measured by the increase in both consumers' surplus and producers' surplus followed by the augmented output production) exceed marginal costs (as measured by both the societal cost of financing the policy and the reduction in the producers' surplus caused by the reduction in the market price), then it is convenient to increase the R&D subsidy to increase welfare. On the contrary, when economic costs outweigh economic benefits, it is convenient to reduce the R&D subsidy to increase welfare. Social welfare is maximised only when marginal benefits equal the corresponding marginal costs. At the optimum, therefore, this policy can eliminate the inefficiencies related to the imperfect appropriability of R&D innovation.

Interestingly and intuitively, the optimal R&D subsidy is decreasing in the extent of the R&D externality. As far as the degree of R&D disclosure increases, the lower the government's need to subsidise a dissemination activity on the producers' side. When disclosure tends to its minimum intensity (i.e., $\beta \to 0$ so that knowledge tends to be a private good), the optimal subsidy is at its highest level. When disclosure reaches its maximum intensity (i.e., $\beta = 1$ so that knowledge is a pure public good), the optimal subsidy is at its smallest level. Of course, there exists an optimal value of the R&D subsidy for each value of the degree of the R&D externality. When $\beta = 0$, firms are indifferent whether to disclose.

3.2. The symmetric subgame in which firms do not disclose (ND/ND)

This section adds another piece to the study of the disclosure decision game by considering the symmetric subgame in which both firms symmetrically choose not to disclose R&D outcomes ($\beta_i = \beta_j = \beta = 0$). By using Eqs. (1)-(4) profits of firm i under ND/ND can be written as follows:

$$\Pi_i^{ND/ND} = (1 - Q)q_i - (w - x_i)q_i - \frac{g}{2}x_i^2, \tag{24}$$

where the upper script ND/ND stands for R&D non-disclosure of both firms. To simplify the narrative, it is sufficient to re-write the relevant equations of the previous section by setting $\beta = 0$. Then, the equilibrium values of the main variables of the problem are those stated in Table 2. In addition, the feasibility conditions of the subgame ND/ND are the following:

$$g > \frac{8}{9} \coloneqq g_{SOC}^{ND/ND} \text{ (second-order condition)}, \tag{25}$$

$$g > \frac{4}{9} \coloneqq g_{SC}^{ND/ND} \text{ (x_i and x_j are strategic substitutes)}, \tag{26}$$

where $g_{SOC}^{ND/ND} > g_{SC}^{ND/ND}$, and

$$g > \frac{4}{9w} \coloneqq g_T^{ND/ND}(w), \text{ (R\&D cost condition)}. \tag{27}$$

Comparison of (26) and (27) reveals that $g_T^{ND/ND}(w) > g_{SC}^{ND/ND}$ for any $w < 1$ and $g_T^{ND/ND}(w) \to g_{SC}^{ND/ND}$ from above for $w \to 1$. Differently, comparison of (25) and (27) reveals that $g_T^{ND/ND}(w)$ can be higher or lower than $g_{SC}^{ND/ND}$ depending on the relative size of w. Proposition 3 deals with this argument for the subgame ND/ND and complements Proposition 1. To this purpose, let us first define $w = \frac{1}{2}$ as a threshold value of the unitary technology of production cost such that $g_T^{ND/ND}(w) = g_{SOC}^{ND/ND}$. Then, the following proposition holds.

Proposition 3. 1) If $w < \frac{1}{2}$ then $g_T^{ND/ND}(w)$ is binding for the subgame ND/ND. 2) If $w > \frac{1}{2}$ then $g_{SOC}^{ND/ND}$ is binding for the subgame ND/ND.

Proof. 1) If $w < \frac{1}{2}$ then $g_T^{ND/ND}(w) > g_{SOC}^{ND/ND}$. 2) If $w > \frac{1}{2}$ then $g_T^{ND/ND}(w) < g_{SOC}^{ND/ND}$. **Q.E.D.**

3.3. The asymmetric subgame in which only one firm discloses (D/ND)

This section continues the analysis made so far by considering the asymmetric subgame in which firm i discloses ($\beta_i = \beta = 0$) and the rival does not ($\beta_j = 0$). By using Eqs. (1), (2) and (3) the profit functions of the disclosing firm i and the non-disclosing firm j read respectively as follows:

$$\Pi_i^{D/ND} = (1 - Q)q_i - (w - x_i)q_i - \frac{g}{2}x_i^2(1 - \beta\sigma), \tag{28}$$

and

$$\Pi_j^{D/ND} = (1 - Q)q_j - \left(w - x_j - \beta x_i\right)q_j - \frac{g}{2}x_j^2, \tag{29}$$

where the upper script D/ND stands for disclosure of the R&D activity of firm i and non-disclosure of the R&D activity of firm j. From Eq. (29) the non-disclosing firm j benefits from the (cost-reducing) externality generated by the stream of knowledge shared by the disclosing firm i, but it does not receive any R&D subsidies by the government as it chose to do not disclose. Differently, Eq. (28) shows that firm i cannot benefit from any cost-reducing activity generated by the R&D effort of the rival, but it receives the R&D subsidy to favour R&D disclosure as it chose to disclose.

At the market stage of the asymmetric game, each firm chooses the optimal amount of output production by maximising its profit. Therefore, the maximisation of (28) and (29) with respect to q_i and q_j, respectively, leads to the following downward-sloping output reaction function of firm i and firm j in the (q_i, q_j) space as a function also of the R&D effort exerted by both firms, that is:

$$\frac{\partial \Pi_i^{D/ND}}{\partial q_i} = 0 \Leftrightarrow \overline{q}_i^{D/ND}(q_j, x_i) = \frac{1 - w - q_j + x_i}{2}, \tag{30}$$

and

$$\frac{\partial \Pi_j^{D/ND}}{\partial q_j} = 0 \Leftrightarrow \overline{q}_j^{D/ND}(q_i, x_i, x_j) = \frac{1 - w - q_i + x_j + \beta x_i}{2}. \tag{31}$$

Eq. (30) and (31) reveal that the output reaction functions of both firms i and j in the asymmetric subgame are similar, differing however in one crucial respect: the non-disclosing firm j can produce more than the disclosing firm i as it is free-riding on the R&D activity generated by the rival without incurring any costs, so that an increase in the R&D externality shift outwards the reaction function of firm j, thereby contributing to increase its production. The solution of the system of output reaction functions $\overline{q}_i^{D/ND}(q_j, x_i)$ and $\overline{q}_j^{D/ND}(q_i, x_i, x_j)$ ($i = \{1,2\}$, $i \neq j$) allows us to get the following equilibrium output obtained by firms i and j at the third stage of the asymmetric subgame D/ND, that is:

$$\overline{\overline{q}}_i^{D/ND}(x_i, x_j) = \frac{1 - w + (2 - \beta)x_i - x_j}{3}. \tag{32}$$

and

$$\overline{\overline{q}}_j^{D/ND}(x_i, x_j) = \frac{1 - w + 2x_j + (2\beta - 1)x_i}{3}. \tag{33}$$

A direct comparison of Eqs. (32) and (33) reveals that the R&D effort of the non-disclosing firm j monotonically reduces production of the disclosing firm i, whereas the R&D effort of the disclosing firm i increases production of the non-disclosing firm j if and only if the R&D externality is high enough ($\beta > \frac{1}{2}$).

Substituting out Eqs. (32) and (33) in the profit equations (28) and (29) allows us to obtain profits of the investing and non-investing firms as a function of the R&D efforts x_i and x_j, i.e., $\Pi_i^{D/ND}(x_i, x_j)$ and $\Pi_j^{D/ND}(x_i, x_j)$ whose maximisation at the second (R&D) stage of the game with respect to x_i and x_j yields a system of R&D reaction functions that can be solved to get the equilibrium R&D investments of firm i and firm j in the asymmetric subgame D/ND, that is:[2]

$$x_i^{*D/ND} = \frac{2(1-w)(2-\beta)(3g-4)}{27g^2(1-\beta\sigma)-6g[4(1-\beta\sigma)+2(2-\beta)^2]+8(2-\beta)}, \tag{34}$$

and

$$x_j^{*D/ND} = \frac{4(1-w)(2-\beta)[3g(1-\beta\sigma)-2(1-\beta)(2-\beta)]}{27g^2(1-\beta\sigma)-6g[4(1-\beta\sigma)+2(2-\beta)^2]+8(2-\beta)}. \tag{35}$$

Eqs. (34) (resp. Eq. (35)) shows that the numerator is positive for any $g > \frac{4}{3}$ (resp. $g > g_{SC}^{D/D,\beta_{low}}(\beta,\sigma)$). Note also that the denominator of both equations is always positive for all the values of g such that the relevant constraints of the disclosure decision game are binding (see Section 3.4).

Like the other subgames, we should augment the analysis by considering the additional constraints on the side of the costs of production of both the disclosing firm i and non-disclosing firm j by explicitly accounting for their R&D cost conditions resulting from the inequalities $w - x_i > 0$ for the disclosing firm i and $w - x_j - \beta x_i > 0$ for the non-disclosing firm j. This can be specialised by substituting out $x_i^{*D/ND}$ and $x_j^{*D/ND}$ from (34) and (35) into the last inequalities showing that the only relevant constraint is the following (coming from the inequality of the disclosing firm i):

$$g > g_T^{D/ND}(\beta,\sigma,w) := \frac{1}{9w(1-\beta\sigma)} \times \Big\{ + \sqrt{\begin{array}{c}\beta^4w^2 - 8\beta^3\sigma w + 16\beta^2\sigma^2w^2 - 6\beta^3w^2 + 24\beta^4\sigma w^2 + 2\beta^3w - 16\beta^2\sigma w + 21\beta^2w^2 \\ -48\beta\sigma w^2 + 10\beta^2w + 32\beta\sigma w - 36\beta w^2 + \beta^2 + 36w^2 - 4\beta - 24w + 4\end{array}} + 6w + 2 - \beta + \beta^2w - 4\beta w\sigma - 3\beta w \Big\}. \tag{36}$$

The inequality in (36) represents the R&D cost condition prevailing in the asymmetric subgame D/ND. Overall, the expressions that determine the feasible region for the emergence of meaningful Nash equilibrium outcomes of the disclosure decision game (that will be studied in Section 3.4) in the (β, g) space are $g > \frac{4}{3}$, $g > g_T^{D/D}(\beta,\sigma,w)$ and $g > g_T^{D/ND}(\beta,\sigma,w)$. When these constraints are binding, the other inequalities (the second-order conditions, the stability conditions and so on) are satisfied accordingly. The shape of the R&D cost conditions depends, amongst other things, on the subsidy rate σ. The analysis of how the shape and position of these constraints change in the (β, g) space is a complex exercise. To avoid lengthening the exposition we leave the complete geometrical analysis to Section 3.4.

Now, substituting out Eqs. (34) and (35) in (32) and (33) allows us to get the equilibrium expressions of q_i and q_j of the asymmetric subgame D/ND, that is:

$$q_i^{*D/ND} = \frac{3g(1-w)(1-\beta\sigma)(3g-4)}{27\ {}^2(1-\beta\sigma)-6g[4(1-\beta\sigma)+2(2-\beta)^2]+8(2-\beta)}, \tag{36}$$

and

[2] The second-order condition are the same as those obtained in the previous sections. Note that we do not report the stability conditions for the asymmetric subgame as there are never binding for the disclosure decision game.

$$q_j^{*D/ND} = \frac{3g(1-w)[3g(1-\beta\sigma)-2(1-\beta)(2-\beta)]}{27g^2(1-\beta\sigma)-6g[4(1-\beta\sigma)+2(2-\beta)^2]+8(2-\beta)}, \tag{37}$$

which are positive for any for any $g > \frac{4}{3}$ and $g > g_{SC}^{D/D,\beta_{low}}(\beta,\sigma)$, respectively.

We can now determine the equilibrium values of the market price and profits of the disclosing firm i and non-disclosing firm j for the asymmetric subgame D/ND, that is:

$$p^{*D/ND} = \frac{9g^2(1-\beta\sigma)(1+2w)-6g\{2(1+w)(2-\beta\sigma)-\beta[1+w(3-\beta)]\}+8(2-\beta)}{27g^2(1-\beta\sigma)-6g[4(1-\beta\sigma)+2(2-\beta)^2]+8(2-\beta)}, \tag{38}$$

$$\Pi_i^{*D/ND} = \frac{g(1-w)^2(1-\beta\sigma)(3g-4)^2[9g(1-\beta\sigma)-2(2-\beta)^2]}{[27g^2(1-\beta\sigma)-6g[4(1-\beta\sigma)+2(2-\beta)^2]+8(2-\beta)]^2}, \tag{39}$$

and

$$\Pi_j^{*D/ND} = \frac{g(1-w)^2(9g-8)[3g(1-\beta\sigma)-2(1-\beta)(2-\beta)]^2}{[27g^2(1-\beta\sigma)-6g[4(1-\beta\sigma)+2(2-\beta)^2]+8(2-\beta)]^2}. \tag{40}$$

The market price in (38) is positive when the relevant constraints of the model, as discussed so far, are fulfilled, whereas $\Pi_i^{*D/ND} > 0$ if and only if $g > g_{SOC}^{D/D}(\beta,\sigma)$ and $\Pi_j^{*D/ND} > 0$ if and only if $g > g_{SOC}^{ND/ND}$. It is important to pinpoint that 1) the public subsidy towards the disclosing firm i monotonically incentivise its R&D effort, production and profits, and 2) if the externality of R&D outcomes of the disclosing firm i towards the non-disclosing firm j is sufficiently small (resp. large), i.e., $\beta < \frac{1}{2}$ (resp. $\beta > \frac{1}{2}$), then the equilibrium values of R&D effort, production and profits of the non-disclosing firm j negatively (resp. positively) depend in a monotonic way on the subsidy rate σ. This means that, if the externality of R&D sharing generated by the disclosing firm i is small, the public provision of R&D subsidies towards disclosure favours the disclosing firm; however, this is detrimental for the non-disclosing rival as the latter cannot adequately benefit from the free-riding activity. Differently, if the externality of R&D sharing generated by the disclosing firm i is large the public provision of R&D subsidies towards disclosure favours both firms, as the non-disclosing rival can benefit adequately from the free-riding activity.

We avoid introducing the equilibrium values of consumers' surplus and producers' surplus for the D/ND subgame (along with the corresponding social welfare function) because, as we will see later, there are no asymmetric Nash equilibria for the disclosure decision game.

The next section studies the owner's choice (at stage one) whether to disclose R&D-related information after being invested in R&D and the government has announced the subsidy policy towards R&D disclosure. Then, it provides a Nash equilibrium analysis along with the corresponding discussion. Section 4 will continue the study of the emergence Nash equilibria of the non-cooperative Cournot duopoly game in which the government first announces and implements the optimal policy discussed in Section 3.1, and then selfish investing firms play the disclosure decision game.

3.4. The disclosure decision stage: Nash equilibria and discussion

This section examines the first stage of the game, in which each firm (after observing the government announcement about the introduction of public subsidies aimed at favouring R&D disclosure) chooses whether sharing R&D outcomes to the rival in a non-cooperative quantity-setting environment à la d'Aspremont and Jacquemin's (1988, 1990).

Making use of the equilibrium firms' profits in (17) for the symmetric subgame D/D, $\Pi^*(0)$ from Table 2 for the symmetric subgame ND/ND, and (39) and (40) for the asymmetric subgame D/ND, it is possible to build on the payoff matrix summarised in Table 3 regarding the Cournot disclosure decision game.

Table 3. The disclosure decision game (payoff matrix).

Firm 2 → Firm 1 ↓	D	ND
D	$\Pi_1^{*D/D}, \Pi_2^{*D/D}$	$\Pi_1^{*D/ND}, \Pi_2^{*D/ND}$
ND	$\Pi_1^{*ND/D}, \Pi_2^{*ND/D}$	$\Pi_1^{*ND/ND}, \Pi_2^{*ND/ND}$

To satisfy the technical restrictions and have well-defined equilibria in pure strategies for every strategic profile (one for each player), the analysis is restricted to the feasibility constraints discussed in Section 3.3 in the (β, g) space, i.e., $g > \frac{4}{3}$, $g > g_T^{D/D}(\beta, \sigma, w)$ and $g > g_T^{D/ND}(\beta, \sigma, w)$, which are assumed to be always satisfied henceforth. Then, to derive all the possible equilibria of the game, one must study the sign of the following profit differentials for $i = \{1,2\}, i \neq j$:

$$\Delta\Pi_a = \Pi_i^{*D/ND} - \Pi_i^{*ND/ND}, \tag{41}$$

$$\Delta\Pi_b = \Pi_i^{*ND/D} - \Pi_i^{*D/D}, \tag{42}$$

and

$$\Delta\Pi_c = \Pi_i^{*ND/ND} - \Pi_i^{*D/D}. \tag{43}$$

We do not explicitly report the expressions of the three equations in (41)-(43) as they are cumbersome and not very informative for our purposes. However, what is important to note in this respect is that their sign may change depending on the main parameters of the model. The solutions of $\Delta\Pi_a = 0$, $\Delta\Pi_b = 0$ and $\Delta\Pi_c = 0$ for g as a function of β and σ allow us to find out the loci of the points that divide the plane (β, g) into areas where the profit differentials are positive from those where they are negative. These solutions are given by $\Delta\Pi_a = 0 \Rightarrow g_a(\beta, \sigma)$, $\Delta\Pi_b = 0 \Rightarrow g_b(\beta, \sigma)$ and $\Delta\Pi_c = 0 \Rightarrow g_c(\beta, \sigma)$. Unfortunately, these expressions cannot be dealt with in a neat analytical form; thus, we cannot present and study them explicitly. To overcome this lacuna, we resort to a geometrical analysis by setting $w = 0.5$, which is nevertheless representative of the general qualitative results of the game emerging for any $0 < w < 1$. The following result summarises the Nash equilibrium outcomes of the disclosure decision game, whose geometrical projection is shown in Figures 1-10, each of them representing the actual shape and position of the relevant constraints of the model (dividing feasible and unfeasible regions) and the loci $g_a(\beta, \sigma)$, $g_b(\beta, \sigma)$ and $g_c(\beta, \sigma)$ in the plane (β, g) for different (increasing) value of σ. To help the reader and clarify the meaning of the figures, we pinpoint that the sand-coloured region represents the unfeasible parameter space and the white region the feasible one. These regions are divided by the R&D cost conditions $g > g_T^{D/D}(\beta, \sigma, w)$ (green solid line) and $g > g_T^{D/ND}(\beta, \sigma, w)$ (red solid line). The y-axis starts at $g = \frac{4}{3}$ in all the cases.

There exist three paradigms: 1) a prisoner's dilemma with a conflict between self-interest and mutual benefit to disclose R&D outcomes following cost-reducing innovation, i.e., the mutually most beneficial action D is dominated by ND; 2) a coordination game in which there are multiple pure-strategy Nash equilibria such that it is mutually beneficial for each player playing the same strategy as the rival does; 3) a deadlock (or anti-prisoner's dilemma) with no conflict between self-interest and mutual benefit to disclose R&D outcomes following cost-reducing innovation, i.e., the mutually most beneficial action D is dominant; 4) a prisoner's dilemma with a conflict between self-interest and mutual benefit to do not disclose R&D outcomes following cost-reducing innovation, i.e., the mutually most beneficial action ND is dominated by D.

Result 1. The Nash equilibrium outcomes of the disclosure decision game are the following.

[1] If $\sigma = 0$ then (ND,ND) is the unique Pareto inefficient Nash equilibrium and the disclosure decision game is a prisoner's dilemma for any $0 < \beta \leq 1$ (Figure 1).

[2] If $\sigma > 0$ then for any $\beta < \frac{1}{\sigma}$ 2.1) (ND,ND) is the unique Pareto inefficient Nash equilibrium and the disclosure decision game is a prisoner's dilemma if β is sufficiently low, and 2.2) (ND,ND) and (D,D) are two pure-strategy Nash equilibria (but D payoff dominates ND) and the disclosure decision game is coordination game if β is sufficiently high (Figure 2).

[3] If σ is set at larger values than those of Point [2] then for any $\beta < \frac{1}{\sigma}$ 3.1) (ND,ND) is the unique Pareto inefficient Nash equilibrium and the disclosure decision game is a prisoner's dilemma if β is sufficiently low, 3.2) (ND,ND) and (D,D) are two pure-strategy Nash equilibria (but D payoff dominates ND) and the disclosure decision game is coordination game for intermediate values of β, and 3.3) (D,D) is the unique Pareto efficient Nash equilibrium and the disclosure decision game is a deadlock if β is sufficiently high (Figure 3, Panels A-D).

[4] If σ is set at larger values than those of Point [3] then for any $\beta < \frac{1}{\sigma}$ (D,D) is the unique Pareto efficient Nash equilibrium and the disclosure decision game is a deadlock (Figure 4).

[5] If σ is set at larger values than those of Point [4] then for any $\beta < \frac{1}{\sigma}$ (D,D) is the unique Pareto efficient Nash equilibrium and the disclosure decision game is a deadlock or, alternatively, (D,D) is the unique Pareto inefficient Nash equilibrium and the disclosure decision game is a prisoner's dilemma (Figure 5, Panels A and B).

The analysis of the disclosure decision game shows that the comparison between the behaviours of selfish firms in a strategic context may have different outcomes resembling several paradigms of the game theory when the government provides public subsidies to incentivise R&D disclosure. To this purpose, Result 1 (whose geometric patterns are represented in Figures 1-5 for different values of σ) clearly implies that the emergence of these outcomes depends on a combination of two parameters representing, respectively, the relative impact on profits of the costs that should be sustained by each firm to install and adopt a cost-reducing R&D technology (g) and the relative weight of the R&D externality (β).

The economic intuition of the different scenarios of the game can be carried out following the narrative of, and the cases detailed in, Result 1. To begin with, we consider the baseline case of no subsidies. Preliminary, we note that the subsidy always increases profits directly as it reduces the cost of R&D effort. However, it also has an indirect equilibrium feedback effect on R&D investments, production and market price. Indeed, it generally tends to increase the R&D effort by allowing for an increase in output (leading to an increase in profits) and a reduction in the market price (which, instead, leads to a reduction in profits). The outcome of the combination of the direct and indirect effects is a priori ambiguous when σ varies. (i) If σ is relatively low, the percentage reduction in the market price is dominated by the percentage increase in output following the increase in R&D investment. Then, the equilibrium feedback effect goes along in the same direction as the direct effect and profits increase. (ii) If σ becomes larger, the percentage reduction in the market price dominates the percentage increase in output following the increase in R&D investment. Then, the equilibrium feedback effect goes in the opposite direction as the direct effect and profits reduce if the latter is larger than the former.

- Case $\sigma = 0$ (Figure 1). The comparison between profits of two profit-maximising innovating firms that must choose whether to disclose supports the action ND irrespective of the parameter scale, and the signs of the three profit differentials are $\Delta\Pi_a < 0$, $\Delta\Pi_b > 0$ and

$\Delta\Pi_c < 0$. The payoff matrix (that can be generated for any pair (β, g) satisfying the relevant constraints of the model) shows that each player would be fully satisfied if he were the only one not to share information on his R&D activity, in turn, free-riding on the R&D disclosure generated by the rival. This allows the former to greatly increase output production at the rival's expense, thus increasing total revenues and profits. However, everyone would prefer to cooperate with the rival by jointly sharing R&D outcomes than giving up information dissemination entirely, to benefit from the increased output production following the mutual and shared cost-reducing innovation activity. Nonetheless, everyone would prefer not to disclose than being the only one suffering from the rival's free riding. This is because the latter scenario would result in the lowest profit due to the smallest output production and total revenues. This situation represents a standard prisoner's dilemma with a conflict between self-interest and mutual benefit to disclose R&D outcomes following cost-reducing innovation. What decisions can we expect from the two selfish and rational players? In this scenario, each player has a dominant strategy (ND) allowing him to get the best outcome regardless of the rival's choices. Therefore, in the absence of subsidies, one can expect that the maximisation of the individual aim will contrast the wishes of Lisbon's Strategy leading to a socially undesirable outcome. Indeed, no one is interested in sharing information about his R&D if the rival is disclosing it. This is because the non-disclosing firm gets the highest possible profit by free-riding on the information shared by the disclosing rival. In addition, no one is interested in disclosure even when the rival is not disclosing to avoid getting the lowest possible profit by undergoing the free-riding activity of the non-disclosing rival. Thus, no one will disclose regardless of the rival's decision so that all players will forego the benefits of information sharing. If both players had decided to collaborate through disclosure, they would be better off. By making decisions that guarantee each player the best outcome unilaterally both are jointly worse off than choosing to disclose. Maximising the individual's will leads to a collective failure, and society is also worse off as producers and consumers would be better off under disclosure. Though the two players are aware that they can achieve this disappointing outcome, in the non-cooperative context we are studying (i.e., without the opportunity to conclude binding contracts), and in the absence of public subsidies, they cannot achieve an agreement allowing disclosure (this is just clear from Eq. (4) by setting $\sigma = 0$, noting also that in this case the cons against disclosure are augmented by the indirect feedback effect that increases rivals' production and profits through free-riding). On one hand, choices are consistent with one another only when players choose not to disclose. Thereafter, no one will regret it as if each of them had unilaterally decided to disclose, he would have favoured the rival at his own expense (allowing him to free ride by increasing both the market size and profits) being, in turn, worse off. On the other hand, choices are not consistent with one another if players had jointly chosen to deviate towards D. Indeed, each player will regret it as if each of them had played ND, he would have increased his profits being, in turn, better off. Therefore, players will be able to avoid disclosure as everyone is aware that no one, will be interested in deviating by playing D if the rival will comply with it. Players, however, will not be able to disclose (non-consistent choices), as everyone knows that no one will be interested in complying with such an agreement if the rival complies with it.

- Case $\sigma = 0.5$ (Figure 2). Introducing public subsidies may change the paradigm of the investment decision game, which is otherwise represented by the prisoner's dilemma. The new paradigms are a coordination game with two pure-strategy Nash equilibria or a deadlock with a Pareto efficient Nash equilibrium depending on the relative size of σ. As Figure 2 shows, to the extent that the subsidy towards disclosure is provided at a relatively low rate, there is a change in the individual incentives of each firm opening the possibility of observing a coordination game depending on the relative size of β and g, also allowing a role for the R&D cost condition $g > g_T^{D/D}(\beta, \sigma, w)$ in addition to $g > \frac{4}{3}$. We first note that, when the R&D activity is relatively efficient ($g \downarrow$), spillovers are less important, so that spreading R&D knowledge does not give rise to a change in the individual incentives to modify players' actions unilaterally (the game is still a prisoner's dilemma), though intense spillovers effects are not viable. This is because combining an intense disclosure with an efficient R&D activity result in "excessive" R&D investment in this strategic setting (see Bacchiega et al., 2010) that, in turn, contributes to reducing $C_i(q_i, x_i, x_j)$ to the extent that the total cost of production of each firm in the symmetric subgame D/D becomes negative. This would increase output and reduce the market price by over-increasing the degree of competition. Differently, for larger values of g, there is room for β to change the individual incentives within the feasible region. When the extent of spill-overs of the R&D investment is relatively low, the subsidy is not effective, and we then observe the same prisoner's dilemma as in the case analysed so far, so that the signs of the three profit differentials are still $\Delta\Pi_a < 0$, $\Delta\Pi_b > 0$ and $\Delta\Pi_c < 0$. When the extent of spill-overs of the R&D investment increases, $\Delta\Pi_b < 0$, so that firm i has an incentive to play D (instead of ND) when the rival is playing D. This changes the nature of the game from a prisoner's dilemma to a coordination game with no dominated strategies – in which every player has the incentive to play the same strategy as the rival – and two pure-strategy Nash equilibria, (D,D) and (ND,ND); however, D payoff dominates ND. The spreading of knowledge substantially increases R&D investments, output and profits when both firms are disclosing. It also reduces (resp. increases) R&D investments, output and profits of the disclosing (resp. non-disclosing) firm in the asymmetric subgames. However, both firms jointly benefit from knowledge spill-overs as the observed percentage increase in profits under D/D is larger than the percentage increase in profits of the non-disclosing free-riding firm in the asymmetric subgames. This is the reason why $\Delta\Pi_b$ becomes negative. Therefore, subsidising R&D to favour disclosure can prevent the pursuit of individual success from causing collective failure by letting cooperation emerge. The payoff matrix, in this case, reveals that players get the best possible outcome if each of them were to disclose. Moreover, both would prefer not to disclose by free-riding on the R&D outcome of the rival's investment activity rather than considering the R&D activity as a pure private good. However, each of them would prefer to abstain from disclosing rather than obtaining the worst possible outcome by being the only one to disclose thus suffering free-riding by the rival. In these circumstances, each player is interested in getting the benefits of disclosure, through a sharp increase in R&D investments and output, if the rival is disclosing to get the best possible outcome. However, no one is willing to disclose if the rival is not disclosing, as no one wants to get the worst possible outcome being the sole suffering rival's free-riding and the reduction in R&D investment, output and profits. Consequently, there are two Nash equilibria: players will not regret if they both disclose (achieving a Pareto efficient outcome) or not disclose (achieving a sub-optimal outcome). As in the prisoner's dilemma, also in the coordination game no rational player wants to make decisions that he might regret. If one of the two players is a risk-taker choosing to disclose to avoid losing the opportunity to increase market share and profits, and the other one is risk-averse choosing to keep his technology secret to avoid being the sole suffering the rival's free-riding activity, both regret their actions as each of them can be better off choosing differently. Therefore, players' decisions are consistent or non-consistent depending on their relative degree of risk aversion. If both players are risk-taker and each of them is willing to be the only one to disclose, the Nash equilibrium is (D,D) and profits are the highest. If both players are risk-averse and each of them is unwilling to be the only one to disclose, the sub-optimal Nash equilibrium is (ND,ND). If one of the two players is risk-averse and the rival risk-taker, decisions will be inconsistent. Unlike in the prisoner's dilemma, however, players now are interested to agree to disclose. Moreover, if they agree to cooperate, each of them is interested to comply with the agreement (even in the absence of binding contracts) as it is not possible to be better off choosing not to disclose. Definitively, the existence of a public subsidy to disclosure generates an incentive towards cooperation.

- Cases $\sigma = 0.9 - \sigma = 1.45$ (Figure 3). Increasing the size of the subsidy increases the complexity of β in determining the outcome of the disclosure decision game, also allowing a role for the R&D cost condition $g > g_T^{D/ND}(\beta, \sigma, w)$. The shape and position of the loci $g_a(\beta, \sigma)$, $g_b(\beta, \sigma)$ and $g_c(\beta, \sigma)$ together with the feasibility constraints are represented in Panels A-D of Figure 3 for different (increasing) values of σ. The figures show that the larger σ the larger the parameter region in which the Nash equilibrium outcome is Pareto efficient, and the game is a deadlock with no conflict between self-interest and mutual benefit to disclose R&D outcomes. This is clear by looking at Panel A ($\sigma = 0.9$) by letting β increase for a given value of g. For small values of β there is a prisoner's dilemma ($\Delta\Pi_a < 0$, $\Delta\Pi_b > 0$ and $\Delta\Pi_c < 0$) as discussed so far. This implies that the intensity of the externality is too low to allow each firm to benefit from knowledge spill-overs. For intermediate values of β there is a coordination game ($\Delta\Pi_a < 0$, $\Delta\Pi_b < 0$ and $\Delta\Pi_c < 0$) with multiple pure-strategy Nash equilibria, as firm i has an incentive to play D (instead of ND) when the rival is playing D. Finally, when knowledge tends to become a pure public good, $\Delta\Pi_a > 0$, so that firm i has an incentive to play D (instead of ND) when the rival is playing ND. This still changes the nature of the game from a coordination game with no dominated strategies to a deadlock with a dominant strategy (D) and a Pareto efficient Nash equilibrium, (D,D). From a strategic point of view a deadlock is less interesting than a prisoner's dilemma. However, this outcome has a relevant policy perspective going along the same trajectory as the Lisbon's Strategy. The continued spreading of knowledge substantially increases R&D investments, output and profits when both firms are disclosing (as expected). However, it eventually *increases* R&D investments, output and profits of the disclosing firm in the asymmetric subgames. This is the reason why $\Delta\Pi_a$ changes the sign becoming positive. Therefore, subsidising R&D at a higher rate can reconcile self-interest and mutual benefit, and the larger σ, the larger the area of Pareto efficiency. In this sense, further increases in the public subsidy makes the extent of the externality of R&D investments less important in determining the outcome of the game.

- Case $\sigma = 1.5$ (Figure 4). When the public subsidy increases further both firms disclose, and the disclosure decision game R&D game becomes a deadlock *irrespective of the parameter scale*. This shows that a public policy may sharply change the individual incentives in a strategic framework going towards a win-win situation (from the point of view of both players and the society). As in the prisoner's dilemma analysed so far, players have a dominant strategy, allowing each of them to get the best outcome regardless of the rival's choices. This strategy, however, is D rather than ND. Each player has a unilateral interest to disclose if the rival is disclosing to get the highest payoff to avoiding getting a worse outcome by not disclosing. Moreover, each player want to disclose even when the rival does not disclose to avoid getting the worst possible outcome if both do not disclose. In this case, the individual behaviour allows to get best outcome for players and the society.

- Cases $\sigma = 2.8$ and $\sigma = 4$ (Figure 5). Finally, larger values of σ contribute to a large increase in the output and a corresponding reduction in the market price to the extent that the percentage reduction in the latter more than compensates for the percentage increase in the former so that profits under D fall short those under ND. For a given g, this happens when the externality of the R&D investment increases. Therefore, too high a subsidy becomes inefficient and the game falls back into the prisoner's dilemma with D (rather than ND) as the dominant strategy.

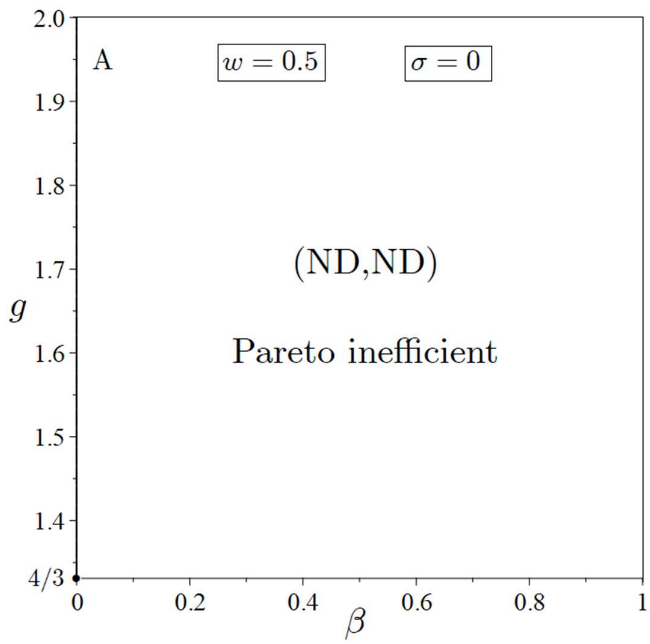

Figure 1. The disclosure decision game when $\sigma = 0$: Nash equilibrium outcomes. The game is a prisoner's dilemma with non-disclosing firms irrespective of the parameter values (area A).

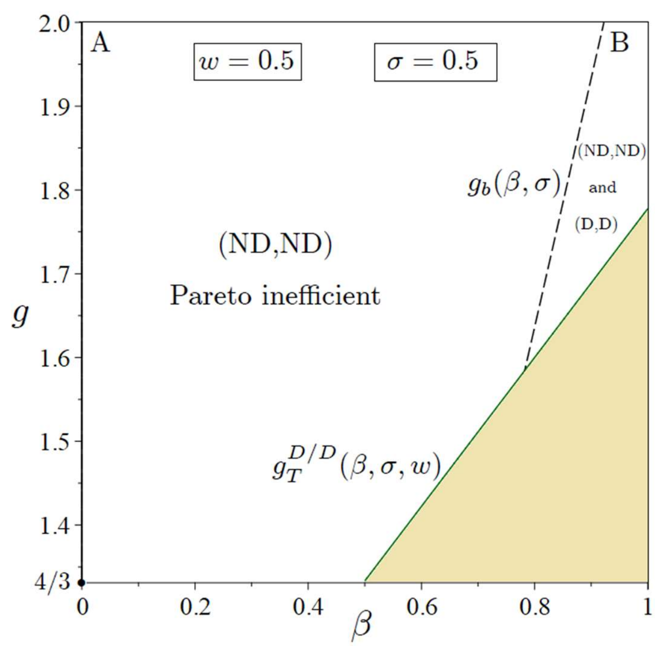

Figure 2. The disclosure decision game when $\sigma = 0.5$: Nash equilibrium outcomes. The sand-coloured region represents the parametric area of unfeasibility. The game can be a prisoner's dilemma with non-disclosing firms (area A) or a coordination game (area B).

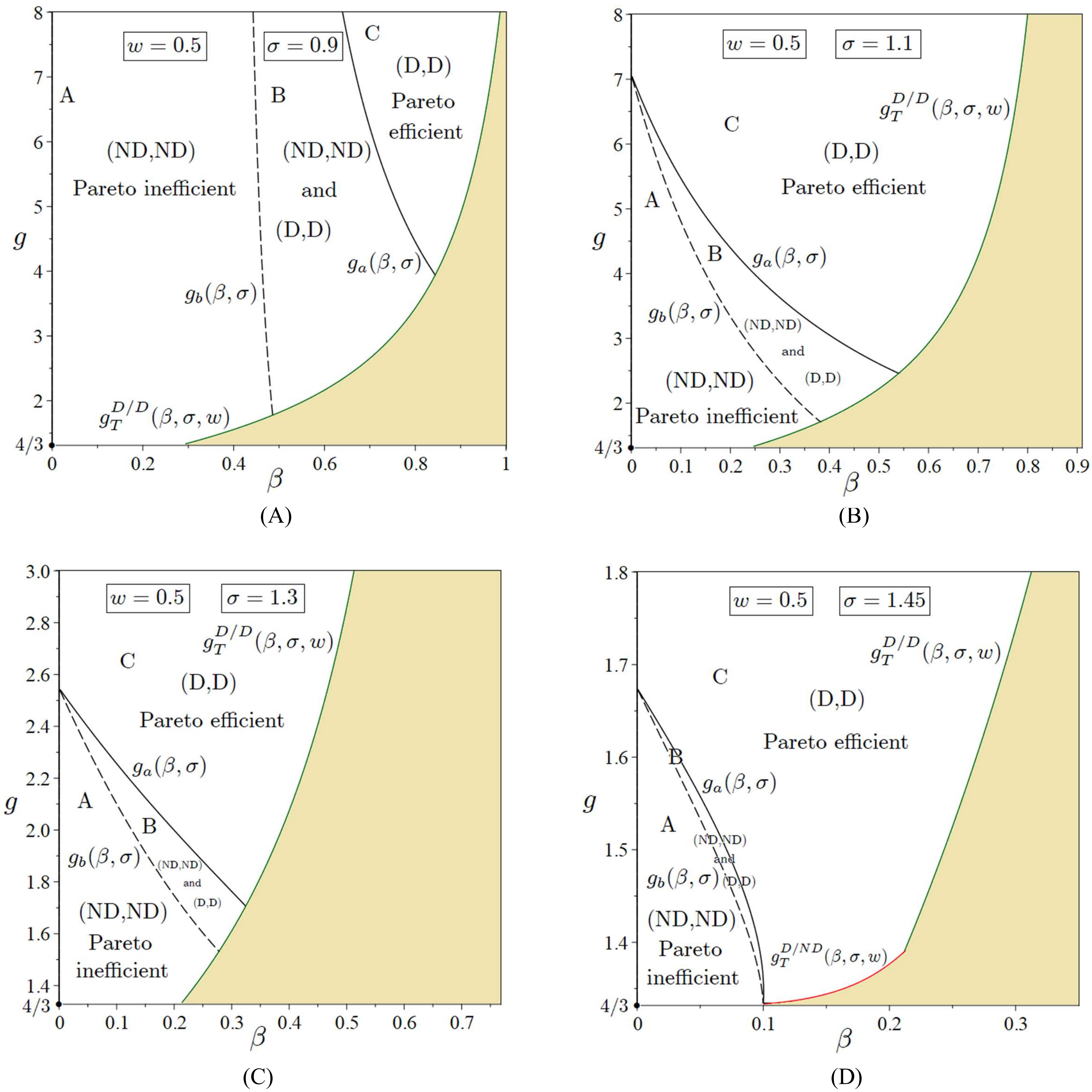

Figure 3. The disclosure decision game when σ ranges from 0.9 (Panel A) to 1.45 (Panel D): Nash equilibrium outcomes. The sand-coloured region represents the parametric area of unfeasibility. The game can be a prisoner's dilemma with non-disclosing firms (area A), a coordination game (area B) or a deadlock with disclosing firms (area C).

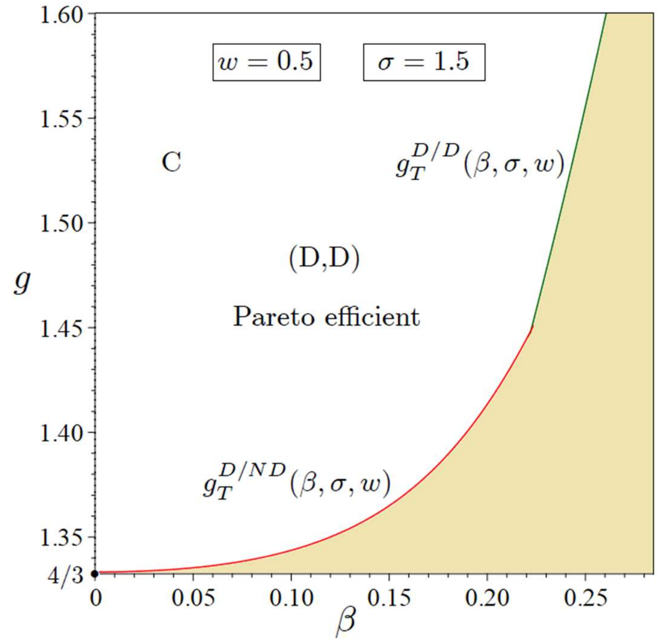

Figure 4. The disclosure decision game when $\sigma = 1.5$: Nash equilibrium outcomes. The sand-coloured region represents the parametric area of unfeasibility. The game is a deadlock irrespective of the parameter values (area C).

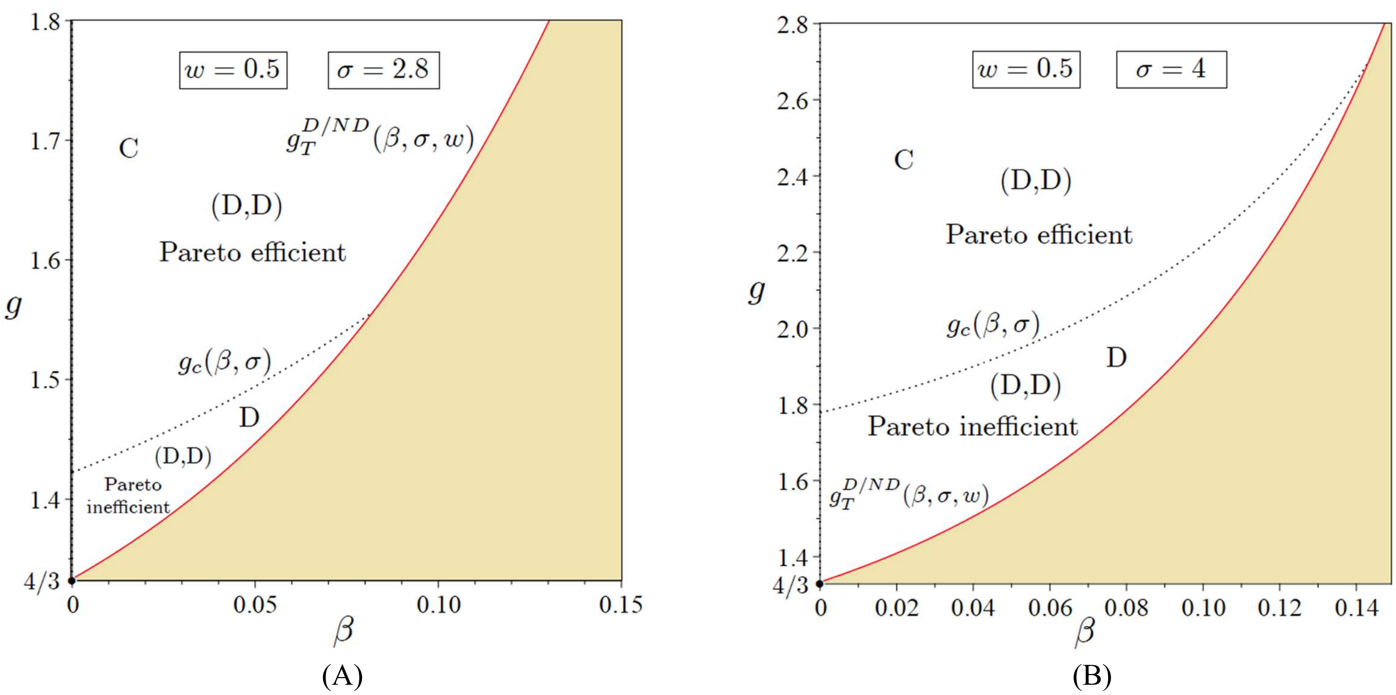

Figure 5. The disclosure decision game when $\sigma = 2.8$ (Panel A) and $\sigma = 4$ (Panel B): Nash equilibrium outcomes. The sand-coloured region represents the parametric area of unfeasibility. The game can be a deadlock with disclosing firms (area C) or a prisoner's dilemma with disclosing firms (area D).

4. R&D disclosure and optimal subsidy

So far, we analysed the disclosure decision game by considering the case in which the government chooses to fund a subsidy to favour R&D knowledge disclosure at any positive exogenous rate $\sigma > 0$ (subject to the feasibility constraint $\sigma < \frac{1}{\beta}$). This subsidy was financed by leaving uniform lump-sum taxes on the side of consumers at a balanced budget (as in Eq. (5)). We now move towards the study of the optimal subsidy discussed in Section 3.1, under which the government maximises social welfare by announcing and implementing $\sigma = \sigma^{OPT} = \frac{3}{2(1+\beta)}$ and then letting firms playing the disclosure decision game at stage one.

Substituting out $\sigma = \sigma^{OPT}$ in the profit equations for the symmetric subgame D/D and the asymmetric subgames, it is possible to state the payoff matrix summarised in Table 4 regarding the Cournot disclosure decision game under optimal policy.

Table 4. The disclosure decision game and optimal policy (payoff matrix).

Firm 2 → Firm 1 ↓	D	ND
D	$\Pi_1^{*\sigma^{OPT}D/D}, \Pi_2^{*\sigma^{OPT}D/D}$	$\Pi_1^{*\sigma^{OPT}D/ND}, \Pi_2^{*\sigma^{OPT}D/ND}$
ND	$\Pi_1^{*\sigma^{OPT}ND/D}, \Pi_2^{*\sigma^{OPT}ND/D}$	$\Pi_1^{*ND/ND}, \Pi_2^{*ND/ND}$

As usual, the analysis is restricted to the feasibility constraints $g > \frac{4}{3}$, $g > g_T^{\sigma^{OPT}D/D}(\beta, w)$ and $g > g_T^{\sigma^{OPT}D/ND}(\beta, w)$ in the (β, g) space. We note that when $\sigma = \sigma^{OPT}$ the inequality $g_T^{\sigma^{OPT}D/ND}(\beta, w) < \frac{4}{3}$ holds for any $0 < \beta \leq 1$ so that only the first and second constraints are binding. The Nash equilibria of the game can be studied considering the sign of following profit differentials for $i = \{1,2\}, i \neq j$:

$$\Delta\Pi_a^{\sigma^{OPT}} = \Pi_i^{*\sigma^{OPT}D/ND} - \Pi_i^{*ND/ND}, \tag{44}$$

$$\Delta\Pi_b^{\sigma^{OPT}} = \Pi_i^{*\sigma^{OPT}ND/D} - \Pi_i^{*\sigma^{OPT}D/D}, \tag{45}$$

and

$$\Delta\Pi_c^{\sigma^{OPT}} = \Pi_i^{*ND/ND} - \Pi_i^{*\sigma^{OPT}D/D}. \tag{46}$$

Unlike the case of exogenous σ, $\Delta\Pi_c^{\sigma^{OPT}} < 0$ irrespective of the parameter scale when $\sigma = \sigma^{OPT}$. Therefore, $\Pi_i^{*\sigma^{OPT}D/D} > \Pi_i^{*ND/ND}$ always holds, whereas the sign of $\Delta\Pi_a^{\sigma^{OPT}}$ and $\Delta\Pi_b^{\sigma^{OPT}}$ can change depending on the relative values of β and g. The following result summarises the Nash equilibrium outcomes of the disclosure decision game under optimal policy, whose geometrical projection is shown in Figure 6 (Panels A-C), along the line of the analysis made so far. The figures are drawn for three different values of the unitary technology of production cost, that is $w = 0.2$ (Panel A), $w = 0.5$ (Panel B) and $w = 0.8$ (Panel C), and aim to help understanding the outcomes of the game showing the shape and position of the constraints along with those of the loci $g_a^{\sigma^{OPT}}(\beta)$ and $g_b^{\sigma^{OPT}}(\beta)$ in the plane (β, g).

Result 2. The Nash equilibrium outcomes of the disclosure decision game under optimal policy are the following depending on the relative values of β and g.

[1] If w is sufficiently low, then: 1.1) (D,D) is the unique Pareto efficient Nash equilibrium and the disclosure decision game under optimal policy is a deadlock; or 1.2) (ND,ND) and (D,D) are two

pure-strategy Nash equilibria (but D payoff dominates ND) and the disclosure decision game is a coordination game (Figure 5, Panel A).

[2] If w is sufficiently high, then: 2.1) (D,D) is the unique Pareto efficient Nash equilibrium and the disclosure decision game under optimal policy is a deadlock; 2.2) (ND,ND) and (D,D) are two pure-strategy Nash equilibria (but D payoff dominates ND) and the disclosure decision game is coordination game; and 2.3) (ND,ND) is the unique Pareto inefficient Nash equilibrium and the disclosure decision game under optimal policy is a prisoner's dilemma (Figure 5, Panel A and Panel B).

Result 2 has a relevant implication showing that – at the optimum – the public provision of a subsidy towards R&D disclosure can modify the unilateral incentives of selfish (profit maximising) firms by sharply changing the scenario prevailing in the market compared to the absence of policies. Though when $\sigma = 0$ the unique Pareto inefficient Nash equilibrium of the game is (ND,ND), the optimal policy leads towards the Pareto efficient outcome (D,D) where it is possible to disclose through the unilateral decisions of rational agents and eventually replicate the case of know-how as a pure public good ($\beta = 1$) if g is relatively high. i.e., obtaining a cooperative-like behaviour with respect to the R&D innovation in a non-cooperative game pairwise maximising social welfare.

We will now briefly comment on the emerging equilibrium outcomes under optimal policy. The higher the efficiency of R&D activity ($g \downarrow$), the lower the need of technological spill-overs. This is because firms, in this case, already incur relatively low production costs, so that disclosure may be detrimental. In this sense, when g is low, the Nash equilibrium is always given by the Pareto efficient outcome (D,D) but only when β is small enough, otherwise the game becomes a prisoner's dilemma. This is because of the sharp increase in profits that each firm can get if it is the sole not to disclose by free-riding on the R&D outcomes generated by the rival. If the efficiency of the R&D activity is lower ($g \uparrow$), knowledge spillovers is convenient so that increasing β – by reducing the size of the optimal subsidy (because of the already high degree of disclosure), in turn needing less resources to finance it – increases R&D, output and profits if both firms are disclosing. However, a large increase in β reduces profits of the disclosing firm below the level it could earn by playing ND when the rival is not disclosing. This happens because of the emerging free-riding activity by the non-disclosing firm. Consequently, for high values of β the game shows no longer a dominant strategy with a Pareto efficient outcome, turning in a coordination game with two pure-strategy Nash equilibria. The lower the efficiency of R&D activity, the higher the unilateral interest of each firm to spill-over its R&D by preserving the Pareto efficient outcome (D,D).

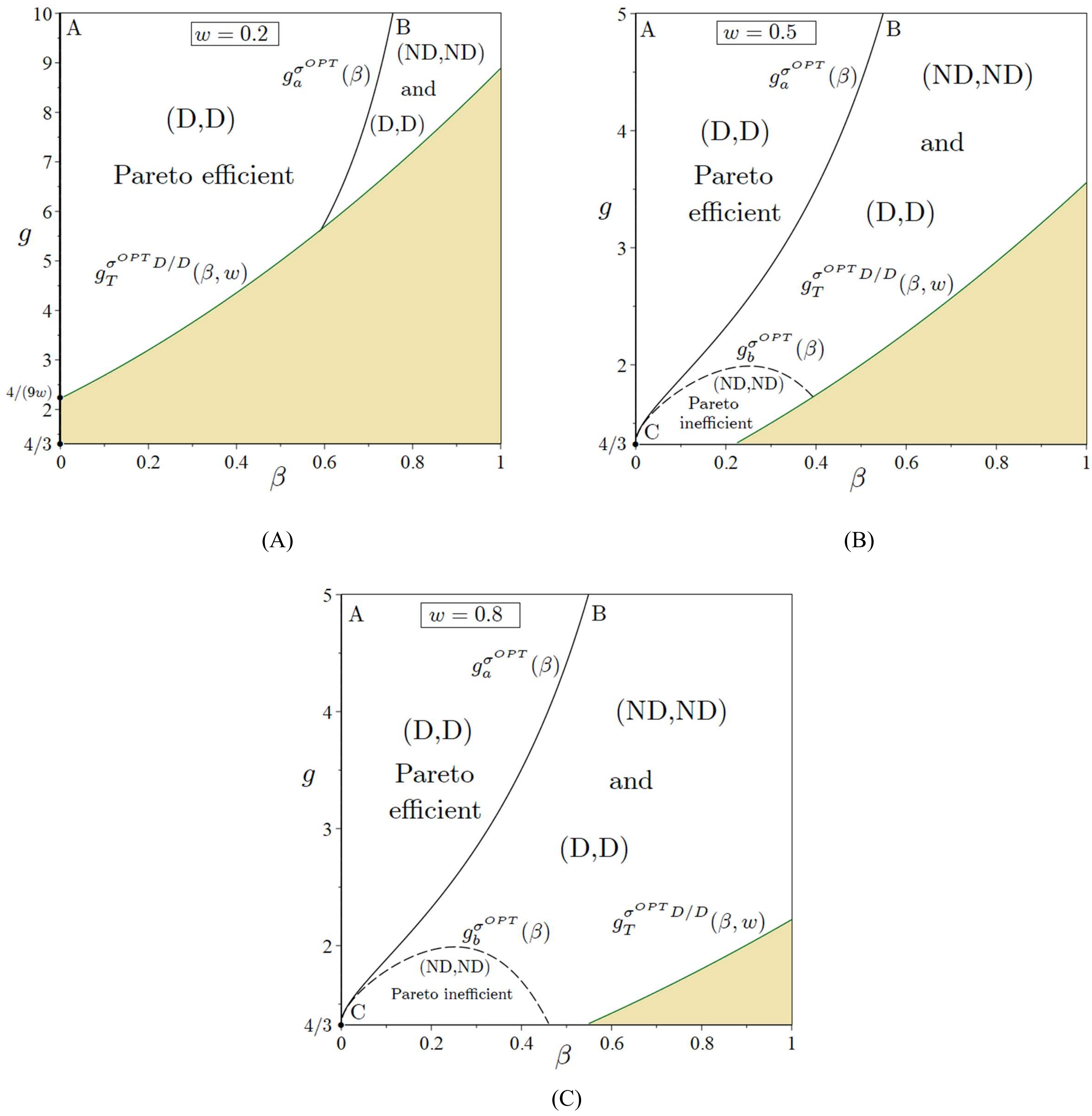

Figure 6. The disclosure decision game when $\sigma = \sigma^{OPT}$ for $w = 0.2$ (Panel A), $w = 0.5$ (Panel B) and $w = 0.8$ (Panel C): Nash equilibrium outcomes. The sand-coloured region represents the parametric area of unfeasibility. The game can be a deadlock with disclosing firms (area A), a coordination game (area B) or a prisoner's dilemma with non-disclosing firms (area C).

5. Conclusions

This work is motivated by the importance of spreading R&D knowledge for the well-being of the society, which is very high on the EU's political agenda, and the impossibility of obtaining this outcome considering the behaviour of selfish, profit-maximising and rational firms performing R&D process innovation due to their strategic incentive to foreclose it. In doing so, the article analyses the firms' strategic choice to disclose or not to disclose when firms invest in R&D and the government designs an ad hoc subsidy to incentivise knowledge disclosure. It contributes to the theoretical literature developed by d'Aspremont and Jacquemin (1988, 1990) about cost-reducing innovation and to the public economics literature providing clear policy insights along the line of Amir et al. (2019) through an optimal policy financed at a balanced budget.

Unlike the received literature on knowledge spill-overs, the article shows that the public authority may change the firm's incentive to share R&D flow in a non-cooperative quantity-setting (Cournot) duopoly pinpointing the emergence of a set of different Nash equilibria. In this regard, the policy can let firms move from a Pareto inefficient non-disclosing outcome to the Pareto efficient disclosure. It represents a win-win result as the game becomes a deadlock (anti-prisoner's dilemma) and social welfare under D/D is larger than social welfare under ND/ND.

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Compliance with ethical standards

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