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Technological Unemployment Revisited: Automation in a Search and Matching Framework

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Technological Unemployment Revisited: Automation in a Search and Matching Framework*

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Abstract

Will low-skilled workers be replaced by automation? To answer this question, we set up a search and matching model that features two skill types of workers and includes automation capital as an additional production factor. Automation capital is a perfect substitute for low-skilled workers and an imperfect substitute for high-skilled workers. Using this type of model, we show that the accumulation of automation capital decreases the labor market tightness in the low-skilled labor market and increases the labor market tightness in the high-skilled labor market. This leads to a rising unemployment rate and falling wages of low-skilled workers and a falling unemployment rate and rising wages of high-skilled workers. In a calibration to German data, we show that one additional industrial robot causes a loss of 1.66 low-skilled manufacturing jobs, whereas the additional robot creates 3.42 high-skilled manufacturing jobs. Thus, overall employment even rises with automation.

Keywords: Unemployment, automation, job search, technological progress, inequality, skill premium.

JEL codes: C78, J63, J64, O33.

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1 Introduction

Reports in the news show on a daily basis that robots are outcompeting humans on more and more tasks (see, for example, The Economist, 2014, 2017; Davison, 2017). This holds true even for tasks that were seen as unautomatable just a few years ago. While industrial robots are substituting for assembly line workers in the automotive industry since decades, recent years are characterized by advances in driverless cars and trucks, diagnosing diseases, producing customized parts and medical implants, writing novels, and even doing science (National Science Foundation, 2009; Schmidt and Lipson, 2009; Barrie, 2014; Abeliansky et al., 2015; Ford, 2015).

Ever since the publication of the working paper version of Frey and Osborne (2013, 2017), who claim that 47% of the jobs in the United States are highly susceptible to computerization over the coming two decades, policymakers, economists, and the general public have been concerned about mass unemployment in the age of automation. However, these high numbers have been criticized for various reasons. For example, Arntz et al. (2016) argue that specific tasks get automated but not whole jobs. They incorporate this insight into the method used by Frey and Osborne (2013, 2017) and calculate that only 9% of all jobs in the United States can be automated in the near future. In addition, there are compensating mechanisms in actually existing economies such as i) decreasing prices of the goods that are produced with robots such that spending on goods and services that are produced by humans might increase, or ii) an increase in the production of robots and 3D printers that – by itself – might require additional human labor.

On top of these arguments, Bloom et al. (2018) take into account that building robots is costly and takes time. Thus, not all jobs that could be substituted by robots from a technical perspective will indeed vanish soon. Instead, economic considerations need to be taken into account: often it will not pay off for firms to substitute cheap labor by expensive robots and the operating costs of robots (in terms of electricity, maintenance, etc.) might also be high as compared to human workers. Bloom et al. (2018) therefore assume a different perspective that is based on the projected growth rate of industrial robots that is determined by the economic environment. They calculate that the predicted evolution of the stock of industrial robots according to the International Federation of Robotics (2017) together with the estimate of Acemoglu and Restrepo (2017) that one industrial robot substitutes for around six workers, implies the automation-driven loss of approximately 60 million jobs worldwide until 2030. Dauth et al. (2017) apply the analysis of Acemoglu and Restrepo (2017) to Germany and find that there, one robot only substitutes for two manufacturing workers. If the numbers of Dauth et al. (2017) are used as a baseline for the calculations, a lower number of jobs will be lost due to automation until 2030 (approximately 20 million worldwide).[1] While these numbers are much less than the 47% of all jobs mentioned by Frey and Osborne (2013, 2017), they are nevertheless large and give rise to some concern. Thus, it is important to analyze the pathways by which robots have the potential to lead to technological unemployment within the modern literature on the determinants of endogenous unemployment levels.

Most recently, Hémous and Olsen (2016) and Acemoglu and Restrepo (2016) pioneered the analysis of the effects of automation on economic growth and inequality within the R&D-based growth literature. In Hémous and Olsen (2016), final goods are produced using a variety of intermediate goods, while in Acemoglu and Restrepo (2016), final goods are produced using a variety of tasks. In both papers the intermediate varieties/tasks are either produced by labor or by labor-replacing machines. R&D-driven innovation leads to new varieties/tasks that always come into existence as un-automated and, thus, have to be produced by human labor. Firms then decide whether to make investments to automate the production of their intermediate variety/task. Along the balanced growth path, there is always a constant range of goods/tasks that are produced by low-skilled workers. As a consequence, technological unemployment is less of a concern in the long run. The wages of low-skilled workers rise due to innovation because a higher rate of creation of intermediate goods/tasks raises the range of these goods/tasks that are produced by low-skilled labor. Even more productive automation could lead to higher wages

[1]These calculations do not take into account that jobs will be created in the manufacturing sector in general equilibrium.

for low-skilled workers because it encourages more innovation. In both contributions, technological unemployment is not at the focus.

As far as the theoretical underpinnings of changing unemployment in the age of automation are concerned, Prettner and Strulik (2017) explore some potential channels. They propose an R&D driven growth model in which new technologies are labor-replacing robots that substitute for low-skilled workers. High-skilled workers are either engineers in the final goods sector or scientists in the R&D sector. Low-skilled workers are employed at assembly lines in the final goods sector. As a consequence, the wages of low-skilled workers stagnate in the face of automation, whereas the wages of high-skilled workers rise. In their model, equilibrium voluntary unemployment will result if there exists a social safety net that is financed out of a wage tax on low-skilled and high-skilled workers. The reason is that the outside option for low-skilled workers becomes more attractive over time because the wages of low-skilled workers stagnate, while the social security benefits rise due to the contributions of high-skilled workers.

In an extension of the model, Prettner and Strulik (2017) show that even involuntary equilibrium unemployment is possible in such a setting. The argument is rooted in the fair wage theory based on Akerlof and Jellen (1990): individuals compare their own wages with those of their peers and perceive their wage as unfair if it lies below a weighted average of their own market clearing wage and the wage of their reference group on the labor market. If workers perceive their wage as unfair, they do not exert full effort at work. The wages of high-skilled workers, which constitute the reference group for low-skilled workers, are higher than the wages of low-skilled workers. Thus, the wages of low-skilled workers that are perceived as fair have to be higher than the low-skilled labor market clearing wages to induce full effort among the workers in this skill group. At this wage rate, more low-skilled workers seek jobs than firms are willing to create. Thus, there is involuntary unemployment of low-skilled workers in equilibrium.

The discussions so far show that equilibrium unemployment in the age of automation can take the form of voluntary unemployment and involuntary unemployment based on fair wage considerations. We aim at contributing to this debate by introducing automation into the modern search and matching theory of frictional unemployment based on Mortensen and Pissarides (1994) and Pissarides (2000). Assuming that low-skilled workers are easier to substitute than high-skilled workers, which is the empirically relevant case up to now, we show that automation leads to higher equilibrium wages of high-skilled workers and to a tighter high-skilled labor market. The reverse holds true for low-skilled workers. As a consequence, unemployment of low-skilled workers rises, while unemployment of high-skilled workers falls. In order to derive quantitative results and to investigate how overall unemployment reacts to automation, we calibrate the model to German data. Surprisingly, it can be shown that overall unemployment even decreases with automation, as fewer low-skilled manufacturing jobs get destroyed than high-skill manufacturing jobs are created. This is consistent with another recent contribution within a search and matching model of employment (Guimarães and Mazeda Gil, 2018) and with empirical evidence showing that more jobs have been created due to automation in major OECD countries than have been destroyed (Autor and Salomons, 2017, 2018; Gregory et al., 2018).

Our paper is organized as follows. In Section 2, we discuss the related literature on automation and search and matching models. Section 3 contains the description of the model. In Section 4, we derive our analytical results, in Section 5 we calibrate the model to German data and simulate the effects of an increase in the stock of industrial robots on low-skilled and high-skilled workers. Finally, in Section 6, we conclude, draw potential lessons for policy makers, and discuss promising future research avenues.

2 Related Literature

Our paper builds upon the literature on automation and the search and matching theory of the labor market. As far as automation is concerned, Steigum (2011) and Prettner (2018) augment the standard neoclassical growth models of Solow (1956), Cass (1965), and Koopmans (1965) by a production factor that is a perfect substitute for labor, while it is accumulated similar to physical capital. They show that this automation capital has the potential to lift an economy out of the traditional stagnation steady state even in the absence of technological progress. The reason is that automation capital makes the production factor labor accumulable such that the Cobb-Douglas production technology is transformed endogenously into an AK production technology. Thus, the possibility for long-run economic growth emerges in the neoclassical growth model, which has considerable consequences for welfare in the long run.

While the long-run implications of capital accumulation for economic growth are strikingly similar in the models of Solow (1956), Cass (1965), and Koopmans (1965) on the one hand, and in the overlapping generations model of Diamond (1965) on the other hand, their implications on the growth effects of automation are the opposite of each other. Sachs and Kotlikoff (2012), Benzell et al. (2015), and Sachs et al. (2015) show numerically that long-run stagnation emerges in an overlapping generations model with automation. Gasteiger and Prettner (2017) provide an analytical explanation for this finding. Since individuals save exclusively out of wage income in the overlapping generations model and automation reduces wages, there is a vicious circle that prevents the economy from taking off. In the standard neoclassical growth framework of Solow (1956), Cass (1965), and Koopmans (1965), by contrast, individuals save out of wage income *and* out of capital income. Thus, a similar vicious circle is not present in these types of models with automation such that long-run growth is feasible.

Irrespective of whether automation is analyzed in the neoclassical growth models of Solow (1956), Cass (1965), and Koopmans (1965) or in the overlapping generations model of Diamond (1965), the distributional effects of automation are similar in this literature. Since automation substitutes for workers but the income of robots flows to capital owners, the capital income share of the economy rises with automation, which is consistent with the stylized facts over the last decades (Elsby et al., 2013; Karabarbounis and Neiman, 2014; Eden and Gaggl, 2018). The fact that wealth is more concentrated than income implies that the automation-induced rise in the capital income share contributes to a rise in overall income inequality (cf. Piketty, 2014; Krusell and Smith, 2015). At the same time, low-skilled workers are still more susceptible to automation than high-skilled workers such that automation leads to a rising skill premium and thereby raises wage inequality (Hémous and Olsen, 2016; Acemoglu and Restrepo, 2016; Prettner and Strulik, 2017; Lankisch et al., 2019).

While there seems to be a consensus that automation will lead to higher inequality, the effects on unemployment are still subject to considerable debates. To gain deeper insights from a theoretical perspective on the endogenous evolution of involuntary unemployment, it is necessary to consider the search and matching model à la Mortensen and Pissarides (1994) and Pissarides (2000). In this type of models, unemployment emerges due to search frictions in the labor market. Assuming such a search and matching based perspective allows us to derive the effects of an increase in the stock of robots on the employment structure via its impact on job creation and the job search behavior of workers that responds endogenously. To our best knowledge, this is the first paper that studies the effects of automation on skill-specific involuntary frictional unemployment.

The contributions of Chassamboulli and Palivos (2013, 2014), Fadinger and Mayr (2014), and Battisti et al. (2017) are related to ours because they use a similar methodological framework. While Fadinger and Mayr (2014) endogenize the state of technology and study the effects of a change in skill endowments, the other articles analyze the impact of skill-specific immigration. All of these articles share important elements with our paper, such as the existence of two separate labor markets, one for high-skilled workers and one for low-skilled workers, and a similar production structure according to which the final good is produced based on a CES production function, while the intermediate goods are produced by high-skilled and low-skilled labor based on a linear technology. The decisive difference to these contributions lies at the level of the exogenous shock. While low-skilled immigration substitutes for low-skilled natives in the production of the low-skill intermediate good in Chassamboulli and Palivos (2013, 2014) and Battisti et al. (2017), automation capital appears in the production function of the final good and substitutes for the low-skilled intensive intermediate good in our paper. Taking taxi drivers as an example, low-skilled immigrants may substitute for low-skilled natives as drivers. However, self-driving cars (automation capital) will be able to replace the occupation group of taxi drivers altogether in the not too distant future. This aspect cannot be analyzed without the presence of the new production factor of automation capital.

Perhaps most closely related to our contribution are the works by Restrepo (2015), Guimarães and Mazeda Gil (2018), and Arnoud (2018). Restrepo (2015) constructs a model of structural unemployment that is due to the skill mismatch that results in the wake of structural change. He shows that – for a severe skill mismatch – unemployment could rise for a prolonged period of time after a recession. This holds for low-skilled and high-skilled workers alike, which discourages skill acquisition overall. With his framework, Restrepo (2015) is able to explain the prolonged high unemployment after the Great Recession. However, he does not consider automation directly in the search and matching model that he proposes. Guimarães and Mazeda Gil (2018), by contrast, propose a search and matching model with automation in which firms choose between traditional technologies and automated technologies. They show that an increase in the productivity of automation raises employment and wages but – at the same time – reduces the labor income share. In addition, they show that the decline in the labor income share is mainly due to technological progress in automation, whereas institutional changes in the labor market only play a rather minor role. In contrast to our contribution, Guimarães and Mazeda Gil (2018) focus on the effects of automation on the labor income share and they abstract from low-skilled and high-skilled workers and from the differential effects that automation can have on these two types of labor. Thus, we view their paper and ours as complementary for the analysis of the effects of automation in the search and matching framework. Finally, Arnoud (2018) sets up a model of wage bargaining with automation. He shows that already the potential access of firms to automation technology raises their bargaining power and thereby their outside option because firms can credibly threaten to substitute workers by robots. This reduction in the bargaining power of workers, in turn, lowers the wages in the economy. Also the framework of Arnoud (2018) abstracts from low-skilled and high-skilled workers. Altogether, our contribution to this strand of the literature is that we show how unemployment and wages are affected differently by automation when distinguishing between low-skilled and high-skilled workers.

3 The Model

In this section, we describe our proposed search and matching labor market model with automation and heterogeneity in the skill level of the workforce. Consider an economy in which workers have two different skill levels $i = \{L, H\}$, where L denotes low-skilled individuals and H denotes high-skilled individuals. The skills are distributed exogenously on a two-point distribution: the fraction λ of the population is low skilled, while the remaining fraction $1 - \lambda$ is high skilled. Normalizing the population size to unity implies that the population shares of a particular skill level are equal to the numbers of low-skilled workers and high-skilled workers, respectively. Time evolves continuously and workers can be in either of two states: employed or unemployed. Workers live indefinitely, are risk neutral, discount the future at the constant rate $r > 0$, and cannot choose to switch their skill level, i.e., education is exogenous and fixed.

3.1 Production Technology

Three goods are produced in the economy. A final consumption good Y and two intermediate goods Y_H and Y_L that are used in the production of the final good. Each high-skilled worker produces one unit of the intermediate good Y_H and each low-skilled worker produces one unit of the intermediate good Y_L. Due to this structure, there is no need to distinguish between the employment level of a given skill type i and the output of the corresponding intermediate good i, thus, $Y_H \equiv H$ and $Y_L \equiv L$. From now on, we refer to the intermediate goods produced by low-skilled workers as low-skilled intensive and to the intermediate goods produced by high-skilled workers as high-skilled intensive.

Apart from high-skilled and low-skilled labor, there are two other production factors: traditional physical capital in the form of machines, assembly lines, and factory buildings, which is denoted by K, and automation capital in the form of industrial robots, self-driving cars, 3D printers, etc. which is denoted by P for "programmable labor." Automation capital is a perfect substitute for low-skilled workers and an imperfect substitute for high-skilled workers (cf. Lankisch et al., 2019). The CES production function of the final good is given by

$$Y = AK^{\alpha}[\gamma(L+P)^{\sigma} + (1-\gamma)H^{\sigma}]^{\frac{1-\alpha}{\sigma}}, \tag{1}$$

where α denotes the elasticity of output with respect to traditional capital, $\gamma \in (0,1)$ refers to the production weight of low-skilled intermediates and of programmable labor, $\sigma \in (-\infty, 1]$ determines the substitutability between both types of workers, and A is an efficiency parameter. From now on, we focus on the empirically relevant case $\sigma \in (0,1)$, in which low-skilled and high-skilled workers are gross substitutes (Autor, 2002; Acemoglu, 2009).

All of the three goods are sold in competitive markets and we use the price of the final good as the numéraire, implying that the prices of the two intermediate goods p_H and p_L are equal to their marginal products. Hence, we have $p_H = \partial Y/\partial H$ and $p_L = \partial Y/\partial L$. Furthermore, firms can buy and sell traditional capital on a competitive capital market without delay. Thus, it holds that $p_K = \partial Y/\partial K = r + \delta$, where δ denotes the depreciation rate of traditional capital. Differentiating Equation (1) and using $p_K = \partial Y/\partial K = r + \delta$, the prices of the two intermediate goods are given by (see Appendix A.1 for the detailed calculations):

$$p_L = (1-\alpha)\gamma A^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{r+\delta}\right)^{\frac{\alpha}{1-\alpha}} \left[(1-\gamma)\left(\frac{H}{L+P}\right)^{\sigma} + \gamma\right]^{\frac{1-\sigma}{\sigma}}, \tag{2}$$

$$p_H = (1-\alpha)(1-\gamma) A^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{r+\delta}\right)^{\frac{\alpha}{1-\alpha}} \left[(1-\gamma) + \gamma\left(\frac{L+P}{H}\right)^{\sigma}\right]^{\frac{1-\sigma}{\sigma}}. \tag{3}$$

If the rate of return on automation capital were lower than the rate of return on traditional capital, rational investors would only invest in traditional capital, and vice versa. For an interior equilibrium to exist, it needs to be the case that both types of investments deliver the same rate of return. Thus, it holds that $p_K = p_P = r + \delta$, with p_P being the price of automation capital.

We immediately see that – for the empirically relevant range of σ – an increase in the number of high-skilled workers increases the price of the goods produced by low-skilled workers and reduces the price of the goods produced by high-skilled workers. In case of an increase in the number of low-skilled workers, the reverse is true. We show later how an increase in P affects the prices of the high-skilled and low-skilled intensive good.

3.2 Labor Market

There are two separate labor markets, one for high-skilled labor and one for low-skilled labor. High-skilled workers direct their job search only to the high-skill intensive sector, while low-skilled workers direct their search only to the low-skill intensive sector (see, for example, Belan et al., 2010; Chassamboulli and Palivos, 2013, 2014; Hagedorn et al., 2016; Battisti et al., 2017; Liu et al., 2017). The matching function of firm i can be formally described by

$$M_i = M(V_i, U_i), \tag{4}$$

where M_i denotes the instantaneous flow of hires, V_i refers to the number of vacancies that are posted, U_i is the number of job-searchers, which equals the number of unemployed workers, and the function $M(\cdot,\cdot)$ exhibits constant returns to scale, is increasing in both arguments, at least twice differentiable, and satisfies the Inada conditions. The arrival rate of any worker per vacancy is $M(V_i, U_i)/V_i \equiv m(\theta_i)$, where $\theta_i \equiv V_i/U_i$ measures the labor market tightness in terms of the number of vacancies per unemployed person in the economy. From these expressions it follows immediately that the arrival rate of any vacancy per unemployed worker is $M(V_i, U_i)/U_i \equiv \theta_i m(\theta_i)$. As a consequence, the arrival rate for firms decreases in θ_i, whereas the arrival rate for workers increases in θ_i.

3.3 Firms

In line with the literature, the firms that produce in the intermediate goods sector are small and each firm offers only one job (see, for example, Mortensen and Pissarides, 1994; Albrecht and Vroman, 2002; Dolado et al., 2009; Gautier et al., 2010). As is well-known, the outcome of the model with single-worker firms is equivalent to a model with large firms that face adjustment costs of employment (Pissarides, 2000). The number of firms is determined endogenously in equilibrium. Firms $i = \{H, L\}$ can either post a high-tech vacancy, which is only suited for high-skilled workers, or a low-tech vacancy, which is only suited for low-skilled workers. The value functions of the firms differ according to whether the firm has filled the vacancy or not. If the firm has filled the vacancy, it produces the corresponding good $i = \{H, L\}$ and sells it on the market for the price p_i. The firm pays the wage w_i to its workers and with a probability $(1 - s_i)$ the vacancy is still filled in the next period, such that s_i is the exogenous rate of job destruction. With this structure it is obvious that the value function of a firm with a filled vacancy is given by

$$r\Pi_i^F = p_i - w_i + s_i(\Pi_i^V - \Pi_i^F). \tag{5}$$

By contrast, a firm that does not fill the vacancy has no labor costs and no revenues but has to pay costs for a vacancy (e.g., job advertisement cost), which are denoted by h_i. With the probability $m(\theta_i)$ the firm manages to fill the vacancy such that its value function is given by

$$r\Pi_i^V = -h_i + m(\theta_i)(\Pi_i^F - \Pi_i^V). \tag{6}$$

3.4 Workers

The behavior of workers can be analyzed in a similar vein as the behavior of firms. Workers who are employed receive the wage w_i and become unemployed in the next instant with the probability s_i. Thus, the value function of an employed worker is given by

$$r\Psi_i^E = w_i + s_i(\Psi_i^U - \Psi_i^E). \tag{7}$$

An unemployed person receives a flow benefit z_i while being unemployed. This flow benefit includes the opportunity costs of employment such as unemployment benefits, leisure, and the potential income generated by home production. With the probability of finding a job being equal to $\theta_i m(\theta_i)$, the value function of an unemployed person is given by

$$r\Psi_i^U = z_i + \theta_i m(\theta_i)(\Psi_i^E - \Psi_i^U). \tag{8}$$

4 Solution of the Model

In this section, we solve the model, describe the steady-state solution, and provide the comparative statics analysis with respect to the effects of the accumulation of automation capital on unemployment and wages of high-skilled and low-skilled workers, respectively.

4.1 Wage Determination

Since the workers strictly prefer being employed to being unemployed and the firms strictly prefer a filled vacancy to the situation of an unfilled vacancy, there is a surplus to be gained from a successful match. We follow the literature and assume that the firm and the worker bargain over the distribution of the surplus from the match in a cooperative bargaining process (see, for example, Mortensen and Pissarides, 1994, 1999; Pissarides, 2000; Gautier, 2002). Once a worker of type i and a firm with the same skill requirements meet each other, they solve a generalized Nash bargaining problem given by

$$\max_{w_i} \left\{ \Psi_i^E - \Psi_i^U \right\}^{\beta} \cdot \left\{ \Pi_i^F - \Pi_i^V \right\}^{1-\beta}, \tag{9}$$

where $\beta \in (0,1)$ represents the bargaining power of the worker. Maximizing the Nash product provides us with the equilibrium expression for the wage rate as given by

$$w_i = z_i + (p_i - z_i) \cdot \Gamma(\theta_i), \tag{10}$$

with

$$\Gamma(\theta_i) = \beta \frac{r + s_i + \theta_i m(\theta_i)}{r + s_i + \theta_i m(\theta_i)\beta}.$$

Thus, the wage is set as a mark-up over the income enjoyed while being unemployed. The mark-up itself consists of two parts. The first part is the profit that a firm earns if it fills a vacancy with an employee who only earns the outside option z_i. This is the highest possible overall profit a firm could make. Second, the term $\Gamma(\theta_i)$ provides the effective bargaining power of the workers as described by Cahuc et al. (2014). This term refers to those part of the highest possible overall profit that a firm can make by filling a vacancy that the workers are able to appropriate by negotiation. As is intuitive, this bargaining power rises with the bargaining weight of the workers (β) and with the labor market tightness (θ), whereas it decreases with the job destruction rate (s_i). Appendix A.2 provides the detailed calculations regarding the derivation of the wage rate.

4.2 Labor Demand and Employment

Firms enter the market and open their vacancies as long as the expected profit of posting a vacancy is positive. Free market entry drives the expected profit of a vacancy down to zero such that

$$\Pi_i^V = 0 \tag{11}$$

holds at the long-run equilibrium. Further, the present value function of a filled job, Equation (5), is used and combined with the equilibrium wage level w_i to obtain the following labor demand:

$$\frac{h_i}{m(\theta_i)} = (1-\beta)\frac{(p_i - z_i)}{r + s_i + \theta_i m(\theta_i)\beta}. \tag{12}$$

Thus, the expected costs of creating a vacancy equal the expected profit of a filled job.

At a steady-state equilibrium, the flows in and out of unemployment have to be equal, i.e. $\dot{U}_i = 0$. Using that the number of low-skilled workers in the economy is given by $\lambda = U_L + L$, with U_L being the number of unemployed low-skilled workers, while the number of high-skilled workers is given by $1 - \lambda = U_H + H$, with U_H being the number of unemployed high-skilled workers, the steady-state unemployment rates u_i are given by

$$\frac{U_L}{\lambda} = u_L = \frac{s_L}{s_L + \theta_L m(\theta_L)} \tag{13}$$

and

$$\frac{U_H}{1-\lambda} = u_H = \frac{s_H}{s_H + \theta_H m(\theta_H)}. \tag{14}$$

Analogously, the employment levels are

$$L = \lambda\frac{\theta_L m(\theta_L)}{s_L + \theta_L m(\theta_L)} \tag{15}$$

and

$$H = (1 - \lambda)\frac{\theta_H m(\theta_H)}{s_H + \theta_H m(\theta_H)}. \tag{16}$$

4.3 Effects of the Accumulation of Automation Capital

Before we start deriving and discussing the central results, we define the steady-state equilibrium of the economy.

Definition 1. *A steady-state equilibrium of our search and matching model with automation and skill heterogeneity is characterized by a stationary economy in which the key endogenous variables* $\left\{\theta_i, p_i, p_k, w_i, H, L, K, u_i\right\}$ *are determined by the following equations:*

(i) the flow Equations (13) *to* (16)*,*

(ii) the prices of the two intermediates as given by Equations (2) *and* (3) *and of capital as given by* $p_k = r + \delta = \alpha Y/K$*,*

(iii) the wage rates as given by Equation (10)*,*

(iv) labor demand for each skill level as given by Equation (12)*.*

Next, we state the central results of our model in the following three propositions. The first proposition describes the effects of automation capital on labor market tightness in the low-skilled labor market and in the high-skilled labor market, respectively.

Proposition 1. *The accumulation of automation capital* P *decreases the labor market tightness in the low-skilled labor market and increases the labor-market tightness in the high-skilled labor market.*

Proof. See Appendix A.3.1 for the formal proof. □

To provide an intuition for this result, we make use of the following lemma.

Lemma 1. *An increase in the stock of robots* P *reduces the price of goods produced by low-skilled workers and raises the price of the goods produced by high-skilled workers.*

Proof. See Appendix A.3.2 for the formal proof. □

An increase in the number of robots leads to a substitution of the goods that are produced with low-skilled labor by robots in final goods production. Thus, the price of the goods produced by low-skilled workers decreases, which leads to lower profits of the firms that produce low-skilled intensive goods. This in turn reduces the number of firms that produce low-skilled intensive goods at the steady-state equilibrium and therefore reduces the overall flow of low-skilled vacancies for a given number of low-skilled workers. Thus, labor market tightness decreases for low-skilled workers. By contrast, the demand of the final goods sector for intermediate goods produced by high-skilled workers increases, which raises the price of high-skilled intensive goods and hence the profits of firms producing these goods. The reason is that H and P are imperfect substitutes, implying that the price of the high-skilled intermediate good depends positively on the amount of automation and negatively on the amount of high-skilled labor. At the steady-state equilibrium, there will then be firm entry into the high-skilled intensive goods production such that the flow of vacancies for a given number of high-skilled workers increases. This, in turn, raises the tightness of the high-skilled labor market.

The second proposition describes the effects of automation capital on the unemployment rates of both types of skills.

Proposition 2. *The accumulation of automation capital P increases the unemployment rate of low-skilled workers and decreases the unemployment rate of high-skilled workers.*

Proof. See Appendix A.3.3 for the formal proof. □

This result is a consequence of the results obtained in Proposition 1. Labor market tightness increases for high-skilled workers as automation progresses, while labor market tightness decreases for low-skilled workers. As a consequence, the job finding probability of high-skilled workers increases, while that of low-skilled workers decreases, which, in turn, lowers the unemployment rate of low-skilled workers and increases the unemployment rate of high-skilled workers.

The third proposition describes the effects of automation capital on the wage rates of both types of workers.

Proposition 3. *The accumulation of automation capital P decreases the wage rate of low-skilled workers and increases the wage rate of high-skilled workers.*

Proof. See Appendix A.3.4 for the formal proof. □

This result is a consequence of the previously obtained results. We have seen already that the marginal product of the low-skilled intensive good in final goods production decreases once that automation is accounted for, while the marginal product of the high-skilled intensive good in final goods production increases. The increase in the price of the goods produced by high-skilled workers leads to a higher match surplus for high-tech firms, which induces vacancy posting and raises labor market tightness in that sector. The so induced increase in the job-finding probability of high-skilled workers improves their outside option and strengthens their bargaining position, which in turn raises their wage rate. For low-skilled workers, the opposite results emerge. This channel is known from the immigration literature, where a similar production function of the final good implies that low-skilled immigrants are perfect substitutes for low-skilled natives and imperfect substitutes for high-skilled natives (see, for example, Chassamboulli and Palivos, 2013, 2014; Chassamboulli and Peri, 2015; Liu et al., 2017).

While our results of the effects of automation on unemployment and wages seem to be intuitive, Propositions 2 and 3 are by no means foregone conclusions. In case of standard models with automation but without the search and matching labor market structure, the effect on high-skilled workers would depend on the elasticity of substitution between low-skilled and high-skilled workers. For a high elasticity of substitution, the wages of high-skilled workers fall in the face of automation (see Lankisch et al., 2019). This is not the case in the search and matching framework because the endogenous general equilibrium forces of firm creation overcompensate for the potentially negative effects of automation on high-skilled worker's wages even for a high elasticity of substitution.

5 Quantitative Results

In this section, we calibrate the model to German data and simulate the effects of an increase in the number of industrial robots per thousand manufacturing workers on low-skilled and high-skilled labor. The quantitative analysis allows us to address the question of whether or not automation will lead to overall job losses.

The calibration uses the following Cobb-Douglas matching function

$$M_i = \xi \cdot U_i^{\kappa} V_i^{1-\kappa}, \tag{17}$$

where ξ denotes the efficiency of the matching process and $\kappa \in (0, 1)$ denotes the matching elasticity. The model is fully characterized by 17 parameters. Table 1 lists nine parameters that are taken from the available empirical literature. First, the elasticity of the matching

Table 1: Baseline Parameter Values

Parameter	Description	Value	Source
κ	Matching elasticity	0.5	Petrongolo and Pissarides (2001)
β	Bargaining power	0.5	Petrongolo and Pissarides (2001)
ξ	Matching efficiency parameter	1	Normalized
A	Production efficiency parameter	1	Normalized
r	Real interest rate	0.049	Chassamboulli and Palivos (2014)
δ	Depreciation rate of traditional capital	0.0508	Prettner (2018)
σ	Elasticity of Substitution	1/3	Autor et al. (1998)
α	Elasticity of traditional capital	1/3	Grossmann et al. (2013)
λ	Share of low-skilled workers	0.74	Battisti et al. (2017)
P	Number of Robots per thousand workers	7.6	Dauth et al. (2017)

function κ is set to 0.5, which is in the range of estimates reported in Petrongolo and Pissarides (2001). Second, following most of the literature, including Petrongolo and Pissarides (2001), the bargaining power β is set to 0.5, so that the Hosios condition ($\kappa = \beta$) is fulfilled (Hosios, 1990). Next, the matching efficiency parameter ξ and the production efficiency parameter A are normalized to unity. The real interest rate r is calculated to be 0.049 (based on the monthly interest rate estimated by Chassamboulli and Palivos, 2014), while the depreciation rate of traditional capital δ is 0.0508 (Bureau of Economic Analysis, 2004; Prettner, 2018). The elasticity of substitution between low-skilled and high-skill intermediate goods σ equals 1/3 (Autor et al., 1998), which is also the value that we attach to the elasticity of traditional capital α (Grossmann et al., 2013). Battisti et al. (2017) estimate the share of low-skilled workers λ to be 0.74. Finally, the number of robots P is reported in Dauth et al. (2017), who estimate 7.6 robots per thousand workers in manufacturing in Germany in the year 2014. The remaining seven parameters of the model are chosen such that the simulation outcomes are consistent with seven calibration targets obtained from German data, see Table 2. Table 3 shows the seven parameters that are obtained by exactly reproducing the number of moments with the model for Germany.

Table 2: Matched Targets

Target	Source	Value
Return to skill for high-skilled workers	EU-SILC	1.45
Low-skill replacement ratio	Hall and Milgrom (2008)	0.71
High-skill replacement ratio	Hall and Milgrom (2008)	0.71
Low-skill vacancy to unemployment ratio	IAB Job Vacancy Survey, EU-LFS	0.2743
High-skill vacancy to unemployment ratio	IAB Job Vacancy Survey, EU-LFS	0.5876
Low-skill unemployment rate	EU-LFS	0.0863
High-skill unemployment rate	EU-LFS	0.0242

Note: All targets are constructed for Germany. All values that are obtained from the EU-LFS and EU-SILC databases refer to the working age population, aged 15-64 or 18-64 (depending on the availability of the data). Further, they are averaged over the period 2011-2015. The vacancy data from the IAB Job Vacancy Survey also covers the years 2011-2015. The skill groups are calculated using the educational attainments of the ISCED-11 classification system. Individuals are low skilled up to secondary school certificate, i.e., up to level 4 of the ISCED scale. Those individuals between levels 5 and 8 of the ISCED scale are high skilled.

Table 3: Calibrated Parameter Values

Parameter	Description	Value
γ	Production weight of low-skilled intermediates and of programmable labor	0.876
h_H	Costs of a high-tech vacancy	0.573
h_L	Costs of a low-tech vacancy	0.759
s_H	High-skill job destruction rate	0.019
s_L	Low-skill job destruction rate	0.049
z_H	Flow income of high-skilled, unemployed workers	0.898
z_L	Flow income of low-skilled, unemployed workers	0.610

Note: Calibrated from moments of the data for Germany.

Our simulation predicts that increasing the number of robots per thousand workers from 7.6 to 8.6 decreases the low-skilled employment ratio by 0.033 percentage points. On the other hand, one more robot increases the high-skilled employment ratio by 0.068 percentage points. If we take into account that the employment share of manufacturing equals 0.198 over the period 2011-2015 in Germany, it can be calculated that one additional robot causes a loss of 1.66 low-skilled manufacturing jobs, whereas one additional robot creates 3.42 high-skilled manufacturing jobs.[2] Thus, and this might be surprising, overall employment even increases with automation after all general equilibrium effects fully play out. This result is qualitatively in line with the analytical results derived by Guimarães and Mazeda Gil (2018) and their numerical findings for a calibration of their model to the United States economy. Our results are also by and large consistent with Dauth et al. (2017) who show that one additional robot destroys two jobs in manufacturing, while the overall net effect is negligible because – due to general equilibrium effects – jobs are created in other parts of the economy, notably in the service sector. Finally, our results are also in line with recent empirical evidence that more jobs have been created due to automation in major OECD countries than have been destroyed (Autor and Salomons, 2017, 2018; Gregory et al., 2018).

[2]The value for the employment share of manufacturing is obtained from the EU-LFS database and refers to working age population, aged 16-64.

6 Conclusions

We use automation capital as an additional production factor and embed it in the standard search and matching model augmented by skill heterogeneity, imperfect substitutability between high-skilled workers and low-skilled workers, and different search costs and job destruction rates across skill levels. In our setting, automation capital is a perfect substitute for low-skilled labor and an imperfect substitute for high-skilled labor. Using this structure, we are able to analyze how an increase in the stock of robots effects wage inequality and involuntary unemployment across skill levels. We show that the accumulation of automation capital decreases the labor market tightness in the low-skilled labor market and increases the labor-market tightness in the high-skilled labor market. This leads to a rising unemployment rate of low-skilled workers and a falling unemployment rate of high-skilled workers. In addition, automation leads to falling wages of low-skilled workers and rising wages of high-skilled workers. These effects are remarkable insofar as they do not depend on the elasticity of substitution between low-skilled and high-skilled workers, which is due to the endogenous firm creation in the model.

In calibrating the model to German data, we show that one additional industrial robot substitutes for 1.66 low-skilled manufacturing jobs, which is largely consistent with the findings on the job destruction due to robots in Germany by Dauth et al. (2017). However, one additional robot also creates 3.42 high-skilled manufacturing jobs. Thus, overall employment even increases with automation after all general equilibrium effects fully play out. This result is in line with the results derived by Guimarães and Mazeda Gil (2018) and with recent empirical estimates (Autor and Salomons, 2017, 2018; Gregory et al., 2018).

Previous contributions have clarified that higher unemployment due to automation could come in the form of i) higher voluntary unemployment if the wages of low-skilled workers stagnate in the wake of automation, while welfare benefits rise with the average wage, and ii) in the form of higher involuntary unemployment if low-skilled workers perceive their wage as unfair and react by exerting less effort. Then firms would need to raise the wages for low-skilled workers above their marginal productivity to induce low-skilled workers to exert full effort. In this situation, equilibrium unemployment would result. In our contribution, we clarify that also higher frictional unemployment might be a result of automation but that this only affects low-skilled workers who can easily be substituted by robots. It is even plausible that the positive job-creation effects on high-skilled workers outweigh the negative effects on low-skilled workers to the extent that overall unemployment falls.

To address the issue of higher wage inequality, it would be desirable from a policy perspective to carefully think about schemes that could compensate the losers in the era of automation without introducing (or, at least, minimizing) negative side effects. To address the issue of higher frictional low-skilled unemployment, investments in education may be appropriate to incentivize more low-skilled workers to participate in higher education and retraining programs.

In the current setting, we abstract from endogenous education decisions such that individuals cannot switch from being low skilled to being high skilled subject to investment costs as in Prettner and Strulik (2017). Introducing such an endogenous education decision could yield additional insights into the long-run adjustment dynamics to rising technological unemployment. In addition, it might be interesting to introduce a service sector in which low-skilled workers could also find work and might not yet be threatened to get replaced by automation capital to a similar extent as in manufacturing (Autor and Dorn, 2013). Another promising avenue for further research would be to augment the search and matching model by fair wage considerations to analyze two distinct sources of involuntary unemployment within the model (cf. Prettner and Strulik, 2017; Kuang and Wang, 2017).

A Appendix

A.1 Prices of the Intermediate Goods

Differentiating the production function, Equation (1), with respect to the number of high-skilled workers, with respect to the number of low-skilled workers, and with respect to traditional capital yields

$$\frac{\partial Y}{\partial H} = p_H = Y(1-\alpha)(1-\gamma)H^{\sigma-1}\Big[(1-\gamma)H^{\sigma}+\gamma(L+P)^{\sigma}\Big]^{-1}, \tag{18}$$

$$\frac{\partial Y}{\partial L} = p_L = Y(1-\alpha)\gamma(L+P)^{\sigma-1}\Big[(1-\gamma)H^{\sigma}+\gamma(L+P)^{\sigma}\Big]^{-1}, \tag{19}$$

$$\frac{\partial Y}{\partial K} = p_K = \alpha A K^{\alpha-1}\Big[(1-\gamma)H^{\sigma}+\gamma(L+P)^{\sigma}\Big]^{\frac{1-\alpha}{\sigma}}. \tag{20}$$

Solving Equation (20) for K and using $p_K = r+\delta$ provides us with

$$K = \left(\frac{\alpha A}{r+\delta}\right)^{\frac{1}{1-\alpha}}\Big[(1-\gamma)H^{\sigma}+\gamma(L+P)^{\sigma}\Big]^{\frac{1}{\sigma}}. \tag{21}$$

Next, we devide Equations (18) and (19) by $p_K = \alpha Y/K$, which can be derived by collecting terms in Equation (20). This yields

$$\frac{p_H}{p_K} = \frac{1-\alpha}{\alpha}(1-\gamma)H^{\sigma-1}K\Big[(1-\gamma)H^{\sigma}+\gamma(L+P)^{\sigma}\Big]^{-1}, \tag{22}$$

$$\frac{p_L}{p_K} = \frac{1-\alpha}{\alpha}\gamma(L+P)^{\sigma-1}K\Big[(1-\gamma)H^{\sigma}+\gamma(L+P)^{\sigma}\Big]^{-1}. \tag{23}$$

Substituting Equation (21) in Equations (22) and (23), using $p_K = r+\delta$, and rearranging leads to the prices of the two intermediate goods as given by Equations (2) and (3):

$$p_L = (1-\alpha)\gamma A^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{r+\delta}\right)^{\frac{\alpha}{1-\alpha}}\left[(1-\gamma)\left(\frac{H}{L+P}\right)^{\sigma}+\gamma\right]^{\frac{1-\sigma}{\sigma}},$$

$$p_H = (1-\alpha)(1-\gamma)A^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{r+\delta}\right)^{\frac{\alpha}{1-\alpha}}\left[(1-\gamma)+\gamma\left(\frac{L+P}{H}\right)^{\sigma}\right]^{\frac{1-\sigma}{\sigma}}.$$

A.2 Wage Determination

Once a worker and a firm with the same skill requirements meet, they bargain over the wage rate. They solve the generalized Nash-bargaining problem given by

$$\max_{w_i} \Big\{\Psi_i^E - \Psi_i^U\Big\}^{\beta} \cdot \Big\{\Pi_i^F - \Pi_i^V\Big\}^{1-\beta}. \tag{24}$$

Maximization of the Nash product delivers the sharing rule

$$\beta[\Pi_i^F - \Pi_i^V] = (1-\beta)[\Psi_i^E - \Psi_i^U]. \tag{25}$$

Using the present value functions, Equations (5) and (7), together with the free entry condition $\Pi_i^V = 0$, the rents of firms and workers can be derived as

$$\Psi_i^E - \Psi_i^U = \frac{w_i - r\Psi_i^U}{r+s_i} \qquad \text{and} \qquad \Pi_i^F - \Pi_i^V = \frac{p_i - w_i}{r+s_i}. \tag{26}$$

Substituting Equation (26) in Equation (25) and rearranging leads to

$$w_i = \beta p_i + (1-\beta) r\Psi_i^U. \tag{27}$$

The wages are the weighted sum of the worker's productivity and the value of unemployment. The weights are given by the bargaining power of the respective participant in the negotiations. Next, $\Psi_i^E - \Psi_i^U$ has to be substituted in the present value function for unemployed workers, Equation (8). For the substitution, the sharing rule (25) is used. This yields

$$\Psi_i^E - \Psi_i^U = \beta \cdot S, \tag{28}$$

with $S = (\Psi_i^E - \Psi_i^U) + (\Pi_i^F - \Pi_i^V)$ being the surplus of a match of the respective bargaining parties. Using Equation (26), it turns out that

$$\Psi_i^E - \Psi_i^U = \beta\Big(\frac{p_i - r\Psi_i^U}{r+s_i}\Big). \tag{29}$$

Substituting Equation (29) in Equation (8), the expected value of being unemployed is

$$r\Psi_i^U = \frac{z_i(r+s_i) + p_i\theta_i m(\theta_i)\beta}{r+s_i+\theta_i m(\theta_i)\beta}. \tag{30}$$

Finally, inserting Equation (30) into Equation (27) and rearranging leads to the wage rate given in Equation (10)

$$w_i = z_i + (p_i - z_i) \cdot \Gamma(\theta_i),$$

with

$$\Gamma(\theta_i) = \beta \frac{r + s_i + \theta_i m(\theta_i)}{r + s_i + \theta_i m(\theta_i)\beta}.$$

A.3 Effects of the Accumulation of Automation Capital P

To see how the wage rates of workers, their employment levels, and their unemployment rates change due to the accumulation of automation capital, it is necessary to derive how the labor market tightness in each labor market is affected by an increase in P. We do this by calculating the total differential of Equation (12) for each labor market and thereby proof Proposition 1 in Section A.3.1. Afterwards, we proof Propositions 2 and 3. However, before we can derive the change in wage rates in Section A.3.4, it is first necessary to derive the change in the prices of the two intermediate goods as given by Equations (2) and (3) and as stated in Lemma 1. Section A.3.2 contains the corresponding proof.

A.3.1 Proof of Proposition 1

Proof of Proposition 1. In the low-skilled labor market we obtain

$$\underbrace{h_L \frac{m(\theta_L)\beta \frac{\partial \theta_L m(\theta_L)}{\partial \theta_L} - [r + s_L + \beta \theta_L m(\theta_L)] m'(\theta_L)}{m(\theta_L)^2}}_{C>0} \frac{d\theta_L}{dP} =$$

$$\left[\frac{dH}{d\theta_H} \frac{d\theta_H}{dP} - \frac{H}{L+P} \left(\frac{dL}{d\theta_L} \frac{d\theta_L}{dP} + 1 \right) \right] \times$$

$$\underbrace{(1-\beta)(1-\alpha)\gamma A^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{r+\delta} \right)^{\frac{\alpha}{1-\alpha}} (1-\sigma)(1-\gamma) \left[(1-\gamma) \left(\frac{H}{L+P} \right)^{\sigma} + \gamma \right]^{\frac{1-2\sigma}{\sigma}} \frac{H^{\sigma-1}}{(L+P)^{\sigma}}}_{B>0}. \tag{31}$$

Rearranging yields

$$\left[C + B\frac{H}{L+P}\frac{dL}{d\theta_L}\right]\frac{d\theta_L}{dP} = B\left[\frac{dH}{d\theta_H}\frac{d\theta_H}{dP} - \frac{H}{L+P}\right]. \tag{32}$$

Analogously, the total differential in the high-skilled labor market is given by

$$\underbrace{h_H\frac{m(\theta_H)\beta\frac{\partial\theta_H m(\theta_H)}{\partial\theta_H} - [r + s_H + \beta\theta_H m(\theta_H)]m'(\theta_H)}{m(\theta_H)^2}}_{D>0}\frac{d\theta_H}{dP} =$$

$$\left[-\frac{L+P}{H}\frac{dH}{d\theta_H}\frac{d\theta_H}{dP} + \left(\frac{dL}{d\theta_L}\frac{d\theta_L}{dP} + 1\right)\right]\times$$

$$\underbrace{(1-\beta)(1-\alpha)\gamma A^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{r+\delta}\right)^{\frac{\alpha}{1-\alpha}}(1-\sigma)(1-\gamma)\left[(1-\gamma)+\gamma\left(\frac{L+P}{H}\right)^{\sigma}\right]^{\frac{1-2\sigma}{\sigma}}\frac{(L+P)^{\sigma-1}}{H^{\sigma}}}_{B_1>0}. \tag{33}$$

Rearranging yields

$$\frac{d\theta_H}{dP} = \frac{B_1}{D + B_1\frac{L+P}{H}\frac{dH}{d\theta_H}}\left[\frac{dL}{d\theta_L}\frac{d\theta_L}{dP} + 1\right]. \tag{34}$$

Substituting Equation (34) in Equation (32) and simplifying yields

$$\frac{d\theta_L}{dP} = -\frac{BD}{C\left(D + B_1\frac{L+P}{H}\frac{dH}{d\theta_H}\right) + BD\frac{H}{L+P}\frac{dL}{d\theta_L}}\frac{H}{L+P} < 0. \tag{35}$$

In the next step, we substitute Equation (35) in Equation (34) to obtain

$$\frac{d\theta_H}{dP} = \frac{B_1 C}{C\left(D + B_1\frac{L+P}{H}\frac{dH}{d\theta_H}\right) + BD\frac{H}{L+P}\frac{dL}{d\theta_L}} > 0. \tag{36}$$

Equations (35) and (36) prove Proposition 1. □

A.3.2 Proof of Lemma 1

Proof of Lemma 1. For the price of the low-skill intensive intermediate good we obtain

$$\frac{dp_L}{dP} = \left[\frac{dH}{d\theta_H}\frac{d\theta_H}{dP} - \frac{H}{L+P}\left(\frac{dL}{d\theta_L}\frac{d\theta_L}{dP} + 1\right)\right] \times \underbrace{(1-\alpha)\gamma A^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{r+\delta}\right)^{\frac{\alpha}{1-\alpha}}(1-\sigma)(1-\gamma)\left[(1-\gamma)\left(\frac{H}{L+P}\right)^{\sigma} + \gamma\right]^{\frac{1-2\sigma}{\sigma}}\frac{H^{\sigma-1}}{(L+P)^{\sigma}}}_{B_2>0}. \tag{37}$$

Inserting Equations (35) and (36) and simplifying yields

$$\frac{dp_L}{dP} = -\frac{B_2CD}{C\left(D + B_1\frac{L+P}{H}\frac{dH}{d\theta_H}\right) + BD\frac{H}{L+P}\frac{dL}{d\theta_L}}\frac{H}{L+P} < 0. \tag{38}$$

The procedure for the price of the high-skill intensive intermediate good is similar. Total differentiation yields

$$\frac{dp_H}{dP} = \left[-\frac{L+P}{H}\frac{dH}{d\theta_H}\frac{d\theta_H}{dP} + \left(\frac{dL}{d\theta_L}\frac{d\theta_L}{dP} + 1\right)\right] \times \underbrace{(1-\alpha)\gamma A^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{r+\delta}\right)^{\frac{\alpha}{1-\alpha}}(1-\sigma)(1-\gamma)\left[(1-\gamma) + \gamma\left(\frac{L+P}{H}\right)^{\sigma}\right]^{\frac{1-2\sigma}{\sigma}}\frac{(L+P)^{\sigma-1}}{H^{\sigma}}}_{B_3>0}. \tag{39}$$

Substituting in Equations (35) and (36) and simplifying provides

$$\frac{dp_H}{dP} = \frac{B_3CD}{C\left(D + B_1\frac{L+P}{H}\frac{dH}{d\theta_H}\right) + BD\frac{H}{L+P}\frac{dL}{d\theta_L}} > 0. \tag{40}$$

Equations (38) and (40) prove Lemma 1. □

A.3.3 Proof of Proposition 2

Proof of Proposition 2. Using the change in labor-market tightness in each labor market, the change in the respective employment levels and unemployment rates, as given by

Equations (13) - (16), can be easily derived as follows:

$$\frac{dL}{dP} = \lambda \frac{s_L}{[s_L + \theta_L m(\theta_L)]^2} \frac{\partial \theta_L m(\theta_L)}{\partial \theta_L} \underbrace{\frac{d\theta_L}{dP}}_{<0} < 0 \tag{41}$$

$$\frac{du_L}{dP} = -\frac{s_L}{[s_L + \theta_L m(\theta_L)]^2} \frac{\partial \theta_L m(\theta_L)}{\partial \theta_L} \frac{d\theta_L}{dP} > 0 \tag{42}$$

$$\frac{dH}{dP} = (1-\lambda) \frac{s_H}{[s_H + \theta_H m(\theta_H)]^2} \frac{\partial \theta_H m(\theta_H)}{\partial \theta_H} \underbrace{\frac{d\theta_H}{dP}}_{>0} > 0 \tag{43}$$

$$\frac{du_H}{dP} = -\frac{s_H}{[s_H + \theta_H m(\theta_H)]^2} \frac{\partial \theta_H m(\theta_H)}{\partial \theta_H} \frac{d\theta_H}{dP} < 0. \tag{44}$$

Equations (42) and (44) prove Proposition 2. □

A.3.4 Proof of Proposition 3

Proof of Proposition 3. Finally, we are able to derive the change in the wage rates. The total differential of the wage rate [as given by Equation (10)] in each labor market is

$$\frac{dw_L}{dP} = \Gamma(\theta_L) \underbrace{\frac{dp_L}{dP}}_{<0} + (p_L - z_L)(1-\beta) \frac{r + s_L}{[r + s_L + \beta \theta_L m(\theta_L)]^2} \frac{\partial \theta_L m(\theta_L)}{\partial \theta_L} \underbrace{\frac{d\theta_L}{dP}}_{<0} < 0, \tag{45}$$

$$\frac{dw_H}{dP} = \Gamma(\theta_H) \underbrace{\frac{dp_H}{dP}}_{>0} + (p_H - z_H)(1-\beta) \frac{r + s_H}{[r + s_H + \beta \theta_H m(\theta_H)]^2} \frac{\partial \theta_H m(\theta_H)}{\partial \theta_H} \underbrace{\frac{d\theta_H}{dP}}_{>0} > 0. \tag{46}$$

This proves Proposition 3. □

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